ON THE REFLECTIVITY OF METALLIC FILMS AT MICROWAVE AND RADIO FREQUENCIES

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On the basis of known formulas it was shown that metallic plates exhibit good reflectivities up to very small thicknesses of about 100 Å, which is a value much smaller than the skin depth at radio frequencies. Reflectivity decreases for thicknesses in the range of a certain characteristic thickness $h \approx 20$ Å which is different for different materials.

1. During the last years much attention has been paid to the problem of applying thin metallic films to radio engineering and, in particular, to microwave engineering. One such application is the use of films as microwave mirrors [1-3] and as large-size film reflectors, for example, of the type of the American inflatable satellite repeater "Echo".

At small thicknesses metallic films show a number of interesting properties thus, for example, almost complete preservation of perfect reflectivity for radio frequencies for thicknesses much less than the skin depth corresponding to the frequency of incident wave. A similar phenomenon described in particular in [1-3] for film thicknesses of the order of 200-300 Å in the wavelength range 4-32 mm was discussed by the authors of the cited works as an anomalous phenomenon not fitting into the modern skin-depth theory (but giving, however, a valuable result from the viewpoint of application).*

In this connection it is of interest to examine in detail the formulas which describe the behavior of an electromagnetic wave incident on a metal.

This work investigates the behavior of the reflection coefficient, absorption coefficient, and the transmission coefficient as a function of the film thickness for normal incidence, essentially for the range of small thicknesses which are of interest.

The examination is limited to the frequency range for which the skin depth $\delta$ exceeds considerably the mean free path $l$ of conduction electrons:

$$\delta \gg l.$$  

(1)

Since for most metals at room temperature the length $l$ is between 100-500 Å (see Table), the results shown below are valid practically for wavelengths $\lambda > 0.1$ mm. Moreover, it is assumed that a film is represented by a solid layer.

2. From the literature (see, for example, [4, 5]) formulas are known for the reflectivities and transmittivities of a wave for the general case of an absorbing layer. From these formulas it is easy to obtain expressions for the amplitudes of reflection $r$ and transmission $p$ for the

* A reader wishing to acquaint himself with the results of [1-3] need read only one of them since the discussion of the results in the three works is almost identical.
specific case of normal incidence of a wave of frequency $\omega$ on a metallic layer of an arbitrary thickness $d$ (Fig. 1) located in free space:

$$r \approx - \frac{\sinh (1 - i) (d / \delta) / \sinh (1 - i) (k\delta + (d / \delta))}{1 + (1 - i) k\delta \cosh (1 - i) (d / \delta)},$$

(2)

$$p \approx (1 - i) k\delta / \sinh (1 - i) (k\delta + (d / \delta)),$$

(3)

where $\delta = c / \sqrt{2\pi\omega}$ is the skin depth at the frequency $\omega$; $c$ is the velocity of light in free space; $\sigma$ is the specific direct-current conductivity of the metal; $k = \omega / c = 2\pi / \lambda$; $\lambda$ is the free space wavelength. The coefficients (in power) of reflection $R$, transmission $P$ and absorption $Q$ are then defined as follows

$$R = |r|^2, \quad P = |p|^2, \quad Q = 1 - R - P.$$  

(4)

The relations (2)--(4) give a complete information on the power and phase of the reflected and transmitted wave for any values of layer thickness.

<table>
<thead>
<tr>
<th>Specific conductivity in a thick specimen, $\sigma$, electrostatic units</th>
<th>Mean free path of electrons in a thick specimen, $l_0$, Å</th>
<th>$h_0 = c / 2\omega_0$ Å</th>
<th>$d_m$, Å</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>$6.12 \cdot 10^{17}$</td>
<td>570</td>
<td>0.78</td>
</tr>
<tr>
<td>Gold</td>
<td>$4.37 \cdot 10^{17}$</td>
<td>410</td>
<td>1.1</td>
</tr>
<tr>
<td>Copper</td>
<td>$5.76 \cdot 10^{17}$</td>
<td>420</td>
<td>0.83</td>
</tr>
<tr>
<td>Aluminum</td>
<td>$3.54 \cdot 10^{17}$</td>
<td>150</td>
<td>1.35</td>
</tr>
<tr>
<td>Sodium</td>
<td>$2.09 \cdot 10^{17}$</td>
<td>350</td>
<td>2.28</td>
</tr>
</tbody>
</table>

Note: The values of the quantities $\sigma$ and $l_0$ are quoted according to [8].

Fig. 1. Incidence of a wave on a metallic layer (medium I and III are vacuums, II is a metal; $d$ is the thickness of the metallic layer).

Fig. 2. Dependences of the modulus of the reflection amplitude $|r| = \sqrt{R}$ (curve 1) and the absorption coefficient $Q$ (curve 2) on the transmission amplitude $p = \sqrt{P}$.

In the case when $d \gg \delta$ we obtain from (2) the following Hagen–Rubens formula for the reflection coefficient of a thick metallic layer:

$$R = 1 - \frac{\sqrt{2\omega / \pi \sigma}}{1 - 2k\delta},$$

(5)
3. The region of small thicknesses were \( d \ll \delta \) is of particular interest. Since in this case

\[
\begin{align*}
\text{sh} \left( (1 - \text{ih}) \left[ (d/\delta) + k\delta \right] \right) & \simeq (1 - \text{ih}) \left[ (d/\delta) + k\delta \right],
\end{align*}
\]

after some simple rearrangements we obtain from (2)-(4) the following:

\[
\begin{align*}
R &= 1 / \left( 1 + (h/d) \right)^2, \quad (6) \\
P &= 1 / \left( 1 + (d/h) \right)^2, \quad (7) \\
Q &= 2 / (\sqrt{h/d} + \sqrt{d/h})^2, \quad (8)
\end{align*}
\]

where \( h = c/2\pi\alpha \), i.e., for films whose thickness is much smaller than the skin depth, the optical characteristics do not depend on frequency (within the limitations imposed by the model of a solid film considered). This should be expected considering that when the thickness of the conductor is less than the skin depth at the maximum frequency used, the current at any lower frequency will be uniformly distributed throughout the whole conductor and then, consequently, the impedance (and, at the same time, also \( R, P, Q \)) will be independent of frequency in contrast to a thick conductor whose impedance is determined not by its total thickness but by the skin depth which is frequency dependent.

In connection with the formula (6) we note one more feature of practical importance. It is considered that in order to obtain a coefficient of reflection \( R \) close to unity, the thickness \( d \) of the metallic layer must not be less than, at least, the skin depth \( \delta \) (for the chosen frequency). However, from (6) it is evident that \( R \) has values very close to unity, even for \( d \ll \delta \), this being related to the very small values of the quantity \( h \) (for good conductors at 0°C and \( d > 100 \, \text{Å} \), \( h \) has a value of the order of 1 Å; see Table). In fact, for example, for silver at a frequency \( f = 3 \cdot 10^8 \, \text{cps} \) (\( \lambda = 1 \, \text{m} \)) the skin depth is equal to \( \delta = 3.5 \, \mu \); but for a silver film of thickness \( 500 \, \text{Å} \) (0.05\( \mu \)) the reflection coefficient of a radio wave is, according to (6), equal to \( R = 0.996 \) (for an aluminum film of the same thickness \( R = 0.994 \)). Therefore, the results obtained in [1-3] can be fully explained within the framework of the usual electromagnetic theory.

The parameters \( R, P \) and \( Q \) depend in a rather complicated manner on the thickness \( d \) (since the specific conductivity \( \sigma \) depends also on \( d \) as shown below). Therefore, a simple relationship (Fig. 2) between these values for \( d \ll \delta \), which follows from (4), (6)-(8), and which does not contain the thickness \( d \) of the layer;

\[
\begin{align*}
\sqrt{R} + \sqrt{P} &= 1, \quad (9) \\
Q &= 2\sqrt{R}P. \quad (10)
\end{align*}
\]

can often be very useful (particularly for an experimental investigation). Formulas (9) and (10) contain not only frequency but also the parameters of the metal itself.

4. When decreasing film thickness to a value of the order of the mean free path of conduction electrons \( l_0 \) in an infinitely thick layer, the mean free path \( l \) in an actual film also decreases; with a further decrease of film thickness, the mean free path is essentially determined not by the interaction with the crystal lattice but by film thickness only (a kind of vacuum state is created for the electron gas which is analogous to a vacuum for ordinary gases when the mean free path of a molecule is only determined by the size of the vessel). Assuming a diffusive reflection of an electron when colliding with a metal surface and ignoring the small-angle scattering of electrons, we can obtain the mean free path of electrons, \( l \), in a metallic film as a function of its thickness \( d \) [6, 7]:

\[
\begin{align*}
l &= \begin{cases} 
  l_0(1 - (l_0/4d)), & d > l_0, \\
  d^2(l_0/4d)^2 + (1/2)\ln(l_0/d)), & d < l_0,
\end{cases} \quad (11)
\end{align*}
\]

where \( l_0 \) is the mean free path of an electron in an infinitely thick specimen (when, for small thicknesses of the order of \( d \sim 100 \, \text{Å} \), the scattering of electrons at the boundaries of crystallites is taken into account, the relation (11) changes into [6]:

\[
l = \frac{f}{2} d(1 + \ln(l_0/d)). \quad (11')
\]

On the other hand, as is known from the theory of electrical conductivity of metals (see, for example, [8]), the specific conductivity of metals is proportional to the mean free path:

\[
\sigma = Ne\mu / \sqrt{2mW_f}, \quad (12)
\]

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where \( N \) is the number of conduction electrons per unit volume; \( e \) is the electronic charge; \( m \) is the electron mass; \( W_F \) is the Fermi level of an electron gas at a given temperature. Hence, using (11), we obtain for the specific conductivity \( \sigma \)

\[
\sigma(d) = \begin{cases} 
\sigma_0 (1 - (\frac{d}{d_0})), & d > d_0, \\
\sigma_0 (d/d_0) ((\frac{d}{d_0}) + (\frac{1}{2}) \ln(\frac{d}{d_0})), & d < d_0,
\end{cases}
\]

(13)

where \( \sigma_0 \) is the specific conductivity of the metal in an infinitely thick layer.

![Fig. 3. Dependences of the reflection coefficient \( R \) (curve 1), the transmission coefficient \( P \) (curve 2), and the absorption coefficient \( Q \) on the thickness of a silver film.](image)

Conductivity \( \sigma \) enters the expressions for optical characteristics through the quantity \( h(d) = c/2\pi\sigma(d) \), which has the dimension of length (the values of \( \sigma_0 \) and \( h_0 = c/2\pi\sigma_0 \) are shown in the Table for some metals). As can be seen from (13), the dependence of the specific conductivity on film thickness is particularly evident in the range of thicknesses smaller than the mean free path of an electron.

Therefore, the relations (6)-(8) together with (13) specify completely the optical characteristics of a film as a function of its thickness (for small thicknesses \( d \ll \delta \)). Figure 3 shows the shape of corresponding curves for silver.

5. When the film thickness decreases, the reflection coefficient \( R \) decreases monotonically, as shown in Fig. 3, and the transmission coefficient \( P \) increases monotonically; however, the absorption coefficient \( Q \) attains maximum value for a certain thickness \( d_m \). It is readily seen from (8) that this occurs for

\[
d_m = h(d_m) = c/2\pi\sigma(d_m),
\]

(14)

and, independently of the properties of the metal, \( Q_{\text{max}} = 0.5 \), and \( R(d_m) = P(d_m) = 0.25 \) [as follows also from (6)-(8)].

Now we shall calculate the thickness \( d_m \). Under consideration of (13), Eq. (14) takes the form

\[
d_m^2 = \frac{2}{(\frac{h_0}{d_0}) + \ln(\frac{d_0}{d_m})}.
\]

Solution of this equation can be represented in the form of an infinite series

\[
d_m = \frac{2\sqrt{h_0}}{\sqrt{3} + \ln(\frac{h_0}{4h_0})(3 + \ln(\frac{h_0}{4h_0})(3 + \ldots))}.
\]

(15)

In the temperature range from about 50 to 500°K (which are the limits for our discussion), the ratio \( t_0/h_0 \) does not fall outside the limits \( 10^{-10} \); in this case the expression (15) can be represented with good accuracy by the approximation:

\[
d_m \approx \sqrt{\frac{l_0h_0}{\sqrt{3.5} / (2 + \ln(2l_0/h_0))}}.
\]
When changing the ratio \( t_0/h_0 \) from 10 to 10\(^4\), the coefficient of \( \gamma_{0}^{-1}h_0 \) changes within the limits from 0.83 to 0.55. However, when (11') is taken into account this coefficient becomes \( \gamma_{3.5}^{-1} \left[ 1 + \ln \left( 2t_0 / h_0 \right) \right] \) which gives limits of its variation from 0.94 to 0.57. It is easy to see that the coefficient of \( \gamma_{0}^{-1}h_0 \) almost does not change in a wide temperature range. In turn, \( h_0 = c / 2\pi \sigma_0 \approx 1 / t_0 \) and the value of \( \gamma_{0}^{-1}h_0 \) do not generally depend on the free path length therefore, on temperature. Therefore, the value of \( d \), which depends very weakly on temperature, is a kind of a characteristic length for a given material. Using (12) we have for \( \gamma_{0}^{-1}h_0 \)

\[
\gamma_{0}^{-1}h_0 = \left( c / 2m \nu \nu(T) / 2\pi N \right)^{1/6}.
\]

In the temperature range of interest the Fermi level \( \nu_{F}(T) \) for an electron gas is approximately equal to the Fermi level at absolute zero [8]:

\[
\nu_{F}(T) \approx \nu_{F}(0) = \left( \frac{\hbar}{2m} \right) (2\pi N N)^{1/6},
\]

where \( \hbar \) is Planck's constant. Hence

\[
\gamma_{0}^{-1}h_0 = \left( c / 2m \nu \nu(T) / 2\pi N \right)^{1/6} \approx 0.7 \left( c / \pi \nu \right)^{1/6} N^{-1/6}.
\]

As is known \( c^2 / \hbar = \alpha = 1/137 \) (fine structure constant), therefore

\[
\gamma_{0}^{-1}h_0 \approx 8.2 N^{-1/6}.
\]

Considering (16) and (17) with an approximation of about 10-20\%, it is possible to assume, taking the coefficient of \( \gamma_{0}^{-1}h_0 \) equal to 0.7, that

\[
d \approx 5.7 N^{-1/6}.
\]

The values of the quantity \( d \) calculated according to (16) for \( T = 0^\circ C \) are shown for some metals in the Table.

6. In connection with the results obtained in Sections 4 and 5 we note the following features:

a) Since the thickness \( d \) for which the absorption coefficient \( Q \) assumes maximum value equal to 0.5, is almost independent of temperature, we can conclude without additional analysis that the shape of the curves \( Q(d) \), \( R(d) \) (and \( P(d) \)) changes very little with temperature.

b) The quantity \( N^{-1/3} \) (\( N \) is the number of conduction electrons per 1 cm\(^3\)) which enters relations (17) and (18), is proportional to the distance between atomic layers; on the basis of these relations we can, apparently, consider that the equivalent number of atomic layers which determines the characteristic thickness \( d \) of a metallic film, does not depend on the metal (within the limits of the solid film model). However, for thicknesses of an order of 20 Å the granularity of the film plays an important role [6, 7] (the critical thickness exceeds 20 Å); therefore, the value of (18) for \( d \) is, apparently, underestimated (still remaining correct in the order of magnitude). In order to refine the value of \( d \) it is necessary to take into consideration the character of the structure of the film and, possibly, any quantum phenomena.

c) In the case when the film thickness \( d \) is much less than the electron free path \( (l_0) \) in a thick layer, the free path length in a film \( (l) \) decreases and assumed values of the order of \( d \) \( (l) \) decreases and assumed values of the order of \( d \) \( [\text{see (11), (11')}] \), i.e., \( l \ll l_0 \), consequently, inequality (1) becomes considerably greater; on the other hand, it becomes greater also by the fact that the calculated value of skin depth \( \delta \) increases with decreasing conductivity \( \nu \) (which is proportional to \( l_0 \), see (12)). In connection with this the frequency range for which the condition (1) is satisfied and a macroscopic approach to the problem is possible, widens considerably into the short-wave range. This, for example, for films of thicknesses about 50 Å the results obtained (6)-(8) and (18) are valid practically for all wavelengths \( \lambda > 0.003 \) cm, i.e., in this case the optical characteristics for far infrared range will be apparently analogous to the characteristics for the microwave range.
7. As was already noted above, the optical characteristics (R, P, Q) of films can be, in the general case, determined from the relations (2)-(4); however, in limiting cases for \(d \gg \delta\) the Hagen–Rubens formula (5) for R and Q is valid, and for \(d \ll \delta\) the relations (6)-(8) are valid into which frequency does not enter.

In connection with this it is of particular interest to discuss the upper limit (with respect to thickness \(d\)) of the applicability of the simplified formulas (6)-(10).

For this purpose we shall use still another approximate expression for R and Q which follows from (2)-(4) and which is valid in the range of thicknesses \(d\) for which \(Q \approx 1 - R \ll 1\) (it is not difficult to see that then both the whole range \(d \gg \delta\) and the intermediate thickness range overlapping with the range \(d \ll \delta\) up to 200–300 Å):

\[
R \approx 1 - Q \approx 1 - \frac{\delta}{\lambda} \left( \frac{\sin(2d/\delta)}{\sin(2d/\delta)} \right)
\]

(19)

(however the transmission coefficient P is a quantity of the second order of small quantities).

For \(d \to \infty\) (19) changes, of course, into (5).

Comparing the values of R and Q given by formula (19) with the values obtained from relations (6) and (8), it becomes evident that relations (6)-(8) give satisfactory results (with an accuracy of the order of several percent for \(Q\) and of the order of 0.001% for \(R\)) for thicknesses of \(d\) up to 0.9–1.0 from the skin depth \(\delta\) for a given frequency.

Therefore, if, for example, we limit the investigation to the incidence of waves, of wavelengths exceeding 1 cm (\(\lambda > 1\) cm), on a silver film, then the formulas (6)-(8) (with (13) taken into account) can be used for thicknesses \(d\) of about 3500 Å (0.35 μ) and smaller (however, in the case of \(\lambda \sim 30\) cm the corresponding limit is about 1.1 μ).

**SUMMARY**

Normal incidence of radio-frequency waves on a thin metallic film was theoretically investigated.

It was shown that for films which are thin compared to skin depth for the maximum frequency employed, the optical parameters do not depend on frequency; then the reflection coefficient remains very close to unity (about 0.9) up to very small thicknesses (about 50–80 Å) which are many times smaller than the skin depth.

When calculating the optical parameters of films having thicknesses less than about 100 Å, it is of essential importance to take into account the dependence of the mean free path of electrons on the film thickness.

The results obtained permit one to conclude that the characteristic length for the reflection, transmission and absorption of a wave in a thin film is not the skin depth but a certain length \(d_m\) independent of frequency and temperature, having a value of about 13–20 Å (depending on the metal) which can be by several orders of magnitude less than the skin depth.

The behavior of the optical parameters of thin films as a function of thickness is almost independent of temperature in the range 50–500°K.

It was also shown that, in the case of films which are thin compared with skin depth, the simplified relations for optical parameters remain valid with high accuracy up to thicknesses of the order of skin depth.

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**REFERENCES**

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