Narrow Line X-Ray Transition Radiation by Electron Beam Traversing a Solid-State Superlattice

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Abstract

Soft x-ray can be generated by low energy electron beams traversing a periodic layered medium with very short spatial period as was shown previously by Kaplan and Datta. We show that a combination of materials with high and low atomic number can produce an intense x-ray radiation with very narrow spectral peaks at the vicinity of K, L, ... atomic absorption edges of each of the materials. We propose selection rules for materials constituting the solid-state superlattice. Our results demonstrate that an inexpensive x-ray source with mega (or submega) electron volt energy of electron beam can be used to generate narrow-line x-ray radiation.

Introduction

When an electron beam traverses the interface between two semi-infinite dielectric materials with different dielectric constants \( \varepsilon_1 \) and \( \varepsilon_2 \), it emits electromagnetic radiation [1]. This phenomenon, which is called transition radiation, is different both from the Cerenkov radiation and from the Bremsstrahlung radiation. This electromagnetic radiation can be greatly enhanced by using a periodic multilayer medium (the so called resonant transition radiation) [2,3] instead of a single interface, see Fig. 1.

The spatial distribution of the intensity of transition radiation then has a conical pattern with most of the intensity at each particular wavelength \( \lambda \) concentrating about some certain angle \( \theta \) of emission measured from the path of the electron beam. Each spatial mode with a number \( r \) has a different resonant angle \( \theta_r \), which is related to wavelength \( \lambda \) and to the speed of electron \( v \) by the resonant condition

\[
\frac{\varepsilon_r^{1/2}}{v} \cos \theta_r = \frac{c}{v} - r \frac{\lambda}{l}
\]

(1)

where \( \varepsilon_r^{1/2} = (\sqrt{\varepsilon_1 + \varepsilon_2})/l \) is the average refractive index, \( l_1 \) and \( l_2 \) are the thicknesses of the individual layers formed by two media, and \( l = l_1 + l_2 \) is the spatial period of the periodic medium. This condition represents the constructive coherent interference of electromagnetic wave at a distance point; this fact was explicitly demonstrated in the recent experiment [4] with a small number of layers.

Usually \([2,3,5-8]\) \( l \gg \lambda \), so that ultra-relativistic electrons with energies 100Mev-50 Gev [5] are required to satisfy condition (1) for real \( \theta_r \). However, with the advent of new technologies, periodic layered structures (and even superlattices) can be constructed with a spatial period less than 100 A'. These structure are widely used as x-ray
mirror [9,10]. It was recently proposed [11-13] to use these structures in order to obtain resonant transition radiation exploiting electron beams with very low energies. In solid-state structure, however, electron scattering and photoabsorption may be the main obstacles to achieving an effective source of x-ray radiation. More recently [14], we studied the losses due to photoabsorption and electron scattering and their effect on resonant transition radiation. We showed that a slightly increase of the electron beam energy will render insignificant the loss of electrons and photoabsorption imposes an energy ceiling above which an increase in electron beam energy produces no significant improvement.

For the transition radiation, electromagnetic field is generated due to the difference in dielectric constant between the neighboring layers and the power of the radiation is proportional to \(\epsilon_1(\omega) - \epsilon_2(\omega)\)^2. Therefore, dispersion of dielectric constants of the materials strongly affect the optimal radiation and the shape of the radiation spectrum. In this paper, we investigate the effect of anomalous dispersion in the dielectric constant on the radiation spectrum, and demonstrated that peaks with narrow spectral width can be generated. Based on this and our previous study on losses due to photoabsorption and electron scattering [14], we propose the rules for selection of materials constituting the solid-state superlattice. Usually, [5,6,8] light elements with small atomic number are used as radiators and air plays a role as a spacer in a system utilizing ultra-relativistic electrons. Since a solid-state structure [12-13] has to be used in order to employ low-energy electron beams and the value of \(\epsilon-1\) is roughly proportional to the number of electrons which increases with atomic number of an atom, heavy elements with large atomic number become appropriate candidates as radiators and light elements as spacers in order to obtain a larger quantity \(|\epsilon_1 - \epsilon_2|\) (at least in a short-wavelength range). The use of materials with large atomic number is further justified by the fact that many [15] absorption lines of these materials fall within the soft x-ray range (0.1 - 2 kev).

Transition radiation in a solid-state superlattice

The differential cross-section for transition radiation including photoabsorption in a multilayer system can be expressed in the following form [2,3,6,8]

\[
\frac{d^2N}{d\Omega d\omega} = F_1F_2F_3
\]  

where \(F_1\) is the differential cross-section for one single interface, \(F_2\) denotes the coherent interference of radiation in a single plate (i.e. in two neighboring interfaces), and \(F_3\) represents the coherence summation of radiation from each layer. The unit of the differential cross-section is number of photon \(N\) per unit solid angle \(\Omega\) in steradian per frequency \(\omega\) per electron. The differential cross-section for a single interface is given by [3]

\[
F_1 = \frac{\alpha(\Delta\varepsilon\beta)^2}{\pi^2\omega} |G|^2
\]  

where \(\Delta\varepsilon = \varepsilon_1 - \varepsilon_2\), \(\alpha = 1/137\) is the fine structure constant, \(\theta\) is the angle of emission, \(\omega\) is the frequency of the radiation, \(\beta = v/c\), and \(G(\beta,\theta)\) is a radiation pattern of a single interface. If the electron energy is lower than that required for Cerenkov radiation (i.e., less than 100 - 500 Mev in our case), \(G(\beta,\theta)\) can be expressed as

\[
G(\beta,\theta) = \frac{\sin\theta(1-\beta^2-\beta\cos\theta)}{2(1-\beta^2\cos^2\theta)(1-\beta\cos\theta)}
\]  

The factor \(F_2\) is attributable to coherent summation of radiation at two neighboring interfaces, and assumes the interference pattern of two sources

\[
F_2 = 4\sin^2(l_2\pi(1/\beta - \sqrt{l_2}\cos\theta)/\lambda)
\]  

where \(l_2\) is the thickness of the denser medium. This expression indicates that the power of radiation in a single plate is four times as large as that of single interface when the interference is constructive.

In order to include both photoabsorption and electron scattering into consideration, we note that both of them affect only "multilayer" factor \(F_3\), whereas the factors \(F_1\) and \(F_2\) remain intact. To include photoabsorption into consideration, we assume that the emitted field decays exponentially as \(e^{-\mu z}/(2\cos\theta)\) where \(\mu = (\mu_1 + \mu_2)/l\) is the average absorption coefficient, \(z\) is the distance traveled by the electron and \(\theta\) is the angle of emission. For \(M\) period, we have

\[
F_3 = \left|\sum_{s=0}^{M-1} \Phi_s\right|^2
\]  

\[
\Phi_s = e^{-(\sigma(M-s)+2\pi N\varepsilon)},
\]  

where \(\sigma = \mu l/(2\cos\theta)\) is dimensionless absorption parameter. The factor of \(\cos\theta\) in \(\sigma\) accounts for the longer path length traveled by the photon due to nonzero angle of emission.

Although both energies and momenta of electrons change as the electrons pass through a layer of the material, most of the transmitted electrons conserve their energies and momenta for films thinner than the mean free path [16]. In the first approximation, we consider only the radiation contribution of these electrons, and describe scattering in terms of the transmission of electron \(T\). Interpreting \(T(z)\) as the probability of an electron to pass through a distance \(z\), we can account for electron scattering by redefining \(\Phi_s\) in Eq. (6) as \(\Phi_s = \sqrt{T(\delta l)}e^{-\sigma(M-s)+2\pi N\varepsilon}\). (The different, less rigorous, approach was used by us previously [17]. The results of [17] depart from the present calculations by less than an order of magnitude.) The dependence of \(T(z)\) on the thickness of the medium, \(z = sl\), can be characterized by the so-called critical length \(L_{cr} = (de/dz)/E_s\), where
\[ \frac{dE}{dz} \] is the energy loss of electrons per path length, and \( E_s \) is the initial electrons energy. \( L_{cr} \) is defined as the length at which virtually all electrons are scattered and absorbed; it can be calculated by using Bethe's formula [18-20]. For \( L_{cr} \) longer than the total length of multilayer structure \( Ml \) (which is usually valid), \( T \) decays almost exponentially, \( T = \exp(-z/L_{cr}) \). It can be shown then that the factor \( F_s \) is expressed as
\[
F_s = e^{-M_{(\sigma+)}}(\cosh(M_{(\sigma)}) - \cos(2MX)/(\cosh(\sigma) - \cos(2X)))
\]
where \( \rho = l/(2L_{cr}) \). Because of the rapid variation of \( F_s \) with respect to angle \( \theta \), it is more meaningful to measure the radiation yield by integrating Eq. (2) over solid angle \( \Omega \) using Eqs. (3), and (7) under the condition that \( r = 1 \) and \( l_1 = l_2 = l/2 \):
\[
\frac{dN}{d\omega} = \int_0^\infty \frac{d^2N}{d\omega d\Omega} d\Omega = \frac{8\beta^2\lambda^2 \sinh(M_{(\sigma)})}{\pi^2 cl}(\cosh(\sigma) - \cos(2X))
\]
Eqs. (7) and (8) reduce to the known respective formulas [5-8] when only photoabsorption is present (i.e. \( \rho = 0 \)).

We now use Eq. (8) to find the optimal geometrical configuration (i.e. number of layers and thickness of each layer) so that the radiation is maximized in the desired frequency range. We maximize the right hand side of Eq. (8) with respect to \( \theta \), and hence obtain \( \theta_0 \) and optimal spatial period \( l_{opt}^{(\sigma+)} \). We then maximize the term \( \sinh(M_{(\sigma)})/\sigma \) with respect to \( M \), and obtain the optimal number of periods as \( M_{opt} = [\ln(\sigma_0^0)/[\beta^2(\sigma_0)]] \). We can now find the maximum radiation with angle \( \theta_0 \), period \( l_{opt} \), and number of period \( M_{opt} \). As a result, an "optimized" Eq. (8) can now be written as
\[
\frac{dN}{d\omega} = \frac{\alpha}{2\pi^2 c} |\Delta \epsilon|^2 \frac{1}{\mu} P Q
\]
where
\[
Q = (1-\beta \lambda/\lambda^2)(1-(1/\beta-\lambda/l)^2)(1/\beta-\lambda/l)
\]
\[
P = (\rho/\sigma_0)^2(\sigma-\rho)
\]

One can see that for sufficiently high electron beam energy (e.g. \( E_s > 1 \) Mev), \( Q_{opt} \) and respective \( l_{opt} \) can be approximated by a very simple formula:
\[
Q_{opt} \approx l_{opt} / \lambda = \gamma^2
\]
For a fixed \( l \), \( Q \) is saturated [5-7] after \( Q_{opt} \) is reached even if electron energy keep on increasing. We discovered, however, that even when the period \( l \) is optimized for each particular \( E_s = (\gamma-1)m_e c^2 \), there is some maximal meaningful energy ("ceiling") of electron beam related now to photo-absorption. Indeed, it is obvious that using a multilayer structure thicker than \( 1/\mu \) will not yield a further increase in radiation. On the other hand, in order to still have a multilayer structure with reasonable number of layers (say M \( \geq 4 \)) [14] the condition for the maximum spatial period of the structure, \( l < 1/4\mu \), must be taken into account. Using this consideration as well as Eq. (11) for \( l = l_{opt} \), we obtain the ceiling of the required \( \gamma \) as \( \gamma < 1/(2\sqrt{\lambda/\mu}) \).

The typical value of \( 1/\mu = 1\mu m \) at \( \lambda \approx 104 \mu \), we obtain \( \gamma \approx 15 \) corresponding to \( E_s \approx 7 \) Mev. For light materials this ceiling may reduce to 2 Mev, and even lower.

The minimum electron beam energy required to obtain appreciable intensity of radiation is achieved when electron scattering results in the same losses as photoabsorption, i.e. \( L_{sc} / \varepsilon = 1/\mu \).

The typical value of \( 1/\mu = 5\mu m \), the minimal electron energy is about 100 kev. Above this energy, photoabsorption begins to dominate over electron scattering.

**Resonances of dielectric constant**

It is readily seen from Eq. (3) that the intensity of transition radiation is directly proportional to \( |\epsilon_1 - \epsilon_2|^2 \) where \( \epsilon_{1,2} \) are respective dielectric constants of the both materials of periodic structure. Photoabsorption coefficient \( \mu \) in Eqs. (7) also affects the intensity of radiation appreciably. Both of these parameters undergo large change near so called atomic absorption edges at which the absorption rises sharply.). In Figs. 2a and 2b the atomic scattering factors \( f_1 \) related to dielectric constant by Eq. (16) below and \( f_2 \) related to absorption by Eq. (17) below versus photon energy are depicted.

Because of the sharp changes in \( f_1 \) and \( f_2 \), one must expect a significant resonant increase of radiation intensity in the vicinity of these atomic absorption edges of either of two materials. For each particular element, these atomic absorption edges (and related resonances of refractive index) are pertinent to the photo-ionization of bounded electrons [21] from their respective atomic shells (K, L, M ...) to the ionization continuum.

The most known effect related to this electron transition, is almost discontinuous jump of absorption \( \mu \) (or \( f_2 \), see below) as the photon energy increases in the vicinity of resonance. The points at which absorption increases almost discontinuously due to photo-ionization of K, L, M or N electrons are called K, L, M, N absorption edges respectively. The most pronounced absorption edges correspond to inner atomic shells. Most of the elements have at least one absorption edge in the soft x-ray range. This situation is depicted in Fig. 3 which shows the location of absorption edge in photon energy versus atomic number for various atomic shells.
b) Fig. 2. Typical behavior of a) atomic scattering factor \( f_1 \) and b) atomic scattering factor \( f_2 \) versus frequency \( \omega \).

The less known (or rather less used) phenomenon is a resonant anomalous dispersion of a real component of dielectric constant at absorption edges, which can result in quite a drastic change of refractive index. For example, although the common notion is that the refractive index \( n \) for x-rays is slightly less than unity, the resonant dispersion at absorption edges can result in \( n \) being significantly greater than 1 [22]. It can readily be noticed though that anomalous dispersion in the x-ray range must have a major importance as far as the Cerenkov and transition radiations are concerned since the behavior of \( n(\omega)-1 \) (in the Cerenkov radiation) or \( n(\omega)-n_2(\omega) \) (in the case of the transition radiation), determines the very existence of these phenomena.

In order to take the anomalous dispersion into account, we use atomic scattering factors to calculate the dielectric constant and absorption.

![Graph](image1)

![Graph](image2)

Fig. 3. Location of absorption edges in photon energy \( h\omega \) versus atomic number \( Z \), with curves K, L, and M corresponding to the respective absorption edges.

The total (complex) dielectric constant can be expressed then in terms of the complex atomic scattering factor

\[
\epsilon = (1-f r_0 \lambda^2 N_A / (2\pi))^2 \quad f = f_1 + if_2
\]

where factors \( f_1 \) and \( f_2 \) are real and can be expressed as [15, 22]

\[
f_1 = Z + \frac{2}{r_c c} \sum \int \frac{\nu^2(\omega^2 - \nu^2)Z_\nu \mu_\nu(\nu) d \nu}{(\omega^2 - \nu^2)^2 + \Gamma_\nu^2 \omega^4}
\]

and

\[
f_2 = \frac{2}{r_c c} \sum \int \frac{\Gamma_\nu \omega^2 \nu^2 Z_\nu \mu_\nu(\nu) d \nu}{(\omega^2 - \nu^2)^2 + \Gamma_\nu^2 \omega^4}
\]

where \( Z_\nu \) is the number of q-shell electrons, \( \mu_\nu \) is the cross-section for photo-ionization of q-shell electrons by a photon with a frequency \( \nu \), \( \omega \) is the frequency of incident photon, \( \Gamma_\nu^2 \) is the damping factor, and \( \omega_c \) is the threshold frequency (i.e. absorption edge frequency). The damping constant \( \Gamma_\nu \) can be determined experimentally. In the ideal case, the damping is due to radiation damping only [22],

\[
\Gamma_\nu = \Gamma_\nu^* = 2 \epsilon_0^2 \omega_0 / (3 m_e c^3) = 2 r_c k_c / 3.
\]

Eqs. (13) and (14) are the counterparts of the Kramers-Kronig relations for atomic scattering factors. Each bound electron in an atom can be treated as an oscillator and the summation of the scattering factors from each electron gives the resulting atomic scattering factor in Eqs. (13) and (14). Far from absorption edges, Eqs (13) and (14) can be reduced to
\[ f_1 = Z + \frac{2}{\tau_0 c} \int_0^\infty \frac{\nu \mu_s(\nu)}{\nu^2 - \nu^2} \, d\nu \]  
(16)

\[ f_2 = \frac{\omega \mu_s(\omega)}{4\pi r_0 c} \]  
(17)

where \( \mu_s = \mu/(N_A) = \sum \mu_s Z_i \) is the atomic cross-section. Since usually \( f_1 \ll f_2 \) and \( 1 << r_0 \lambda^2 N_A f_1/(2\pi) \), we can further simplify Eq. (12) as

\[ \epsilon \approx 1 - f_1 r_0 \lambda^2 N_A \pi \]  
(18)

With experimental and theoretical data for atomic scattering factors in Ref. [15], we can readily calculate absorption coefficient and dielectric constant with Eqs. (16) and (17), and make the following observations. As expected, we find \( f_1 \approx Z \) at the frequencies higher than the frequency of K absorption edge since all the electrons can then be considered as free. Elements with \( Z = 1 \) to 4 have their K absorption edges below 0.2 kev, Fig. 3. However, there are many absorption edges of elements of high \( Z \) falling within the soft x-ray range as depicted in Fig. 3. As a result, Eq. (18) has to be used for calculation of dielectric constant which can be larger than unity around the absorption edges owing to anomalous dispersion.

Unfortunately, the known experimental data (see e.g. Ref.[15]) do not provide any significant information on the atomic scattering factor \( f_1 \) in the very close vicinity of absorption edges, i.e. in the range being of the most interest for our purposes. Apparently this is related basically to the fact that so far, there were no immediate practical needs for this kind of data. Therefore, in order to obtain the data on the resonances of real part of refractive index, we have to make a theoretical estimates based on Eq. (13) using some reasonable assumptions. Assuming, in a rough approximation, in Eq. (13) [15,22,23]

\[ \mu_{eq} = \frac{r_0 c}{2\omega_q} \left( \frac{\nu}{\omega_q} \right)^p \]  
(19)

where increment \( p \) varies [22,23] for different shells (and frequencies) remaining basically between \( p = 3 \) and \( p = 2 \), we rewrite Eq. (13) in the form

\[ f_1 = \sum Z_i J_i \]  
(20)

where

\[ J_i = (p_i - 1)s^2 \int_1^\infty \frac{(z^2 - s^2)^2}{[(z - s)^2 + \Gamma_0^2 z^4]} \, dz \]  
(21)

where \( s = \omega/\omega_q \). Using \( p = 3 \) which is approximately valid for K-electrons [23], one readily obtains that in this case

\[ J_K = 1 + \frac{1}{4\pi} \ln \left( \frac{(z - 1)^2 + \Gamma_0^2 /4}{(z + 1)^2 + \Gamma_0^2 /4} \right) \]  
(22)

whereas for \( p = 2 \) (which is approximately the case for L-electrons)

\[ J_L = 1 + \frac{1}{2\pi^2} \ln \left( \frac{(z - 1)^2 + \Gamma_0^2}{1 + \Gamma_0^2} \right) \]  
(23)

For M-electrons \( p \) is close [24,25] to 2.5; calculations for this and some other fractional quantities of \( p \) with \( 2 < p < 3 \) [which result in much more complicated functions compared with Eqs. (22) and (23)] can be found in Ref.[22]. At the present stage, however, it does not seem justifiable to go into too detailed picture of the resonances, since \( p \)-factor in general depends not only on the kind of shell, but also on frequency, and, quite possible, on many other factors. Therefore, in our further calculations in this paper, for all shells other than K-shell we will use Eq. (23) for \( p = 2 \), which results in slightly reduced quantity for the magnitude of \( f_1 \), and therefore, in reduced maximum intensity of radiation.

All solid elements have sharp resonance at K-edge, beginning from \( Z = 3 \), i.e. from lithium, whose K-absorption edge corresponds to \( \lambda_K \approx 226.5 \AA \); \( \lambda_K \) decreases as \( Z \) increases. Since the photo-ionization gives rise to sharp resonances only when electrons are excited from the inner atomic shells, the L-shells can be used beginning from \( Z = 14 \) (Silicon, with the L-absorption edge at \( \lambda_L \approx 123 \AA \)), M-shells beginning from \( Z = 48 \) (cadmium, with \( \lambda_M \approx 28,13 \AA \)), and N-shells beginning from \( Z = 58 \) (cerium with \( \lambda_N \approx 100 \AA \)); in this particular case, the N-shell resonance according to data [15] becomes very sharp even before N-shell is completely filled up). For all these shells and elements, the factor \( f_1 \) at the absorption edge, according to Eqs. (20) - (23), can approximately be written as

\[ f_1 \approx Z - A_q + B_q \left( J_q \left( \omega/\omega_q \right)^-1 \right) \]  
(24)

where subscript \( q \) labels the type of shell (K, L, M, N), and \( A_q = 0, 2, 10, 28 \) and \( B_q = 2, 8, 18, 32 \), for the K, L, M and N shells respectively. The Eq. (24) and Eqs. (20) - (23) were used by us to calculate parameters of the system for a few specific examples of materials. Although the increment \( p \) is not very well defined for L, M and N shells, the total error in the peak intensity of radiation with \( p = 2 \) does not exceed 10 - 20%. The important factor that needs further detailed investigation, is the splitting of resonance for the higher shells (L, M, N) into sub-shell levels, which should reduce somehow the resonance peak of \( f_1 \) and therefore, the peak intensity of the transition radiation. It must be noted though that all the results using Eqs. (22) and (24) should be valid for K-shell lines. One can also note, that the peak resonant magnitude of \( f_1 \) determined by Eqs. (22) or (23) is very insensitive to the variation of damping parameter.
\( \Gamma \) compare with radiation damping Eq. (15), since this magnitude at the point of resonance (\( z = 1 \)) depends on \( \Gamma \) as \( \approx \ln(\Gamma) \).

Selection of materials

Proposed method of x-ray generation is suitable to obtain radiation in the entire range of wavelengths 1 - 200 A. Using various combinations of materials, a broad spectrum of x-ray radiation can be obtained. This method, however, can be envisioned as the most efficient way of generation of narrow x-ray lines radiation in the vicinity of absorption edges of various elements where dielectric constant \( \varepsilon \) varies drastically. The idea is that for each desired frequency the alternating layers are chosen in such a way that one of the materials of the multilayer structure has its resonant frequency (i.e., frequency of one of its absorption edges) in the vicinity of the desired frequency. We will call such material (or layer) a "radiator". The other material must be chosen in such a way that its absorption edges are far from the chosen frequency of radiation. We will call such layers "spacers". Another requirement for the spacer is that its absorption in the vicinity of desired frequency be minimum. This brings us to the condition that for the most of frequencies of radiation the spacer must be a light element with a low atomic number. Yet another "technological" requirement is for each material to form stable layers with smooth surfaces. All these requirements single out three light elements ("island of light elements"): B, Be, and C as the best candidates. It is worth noting that carbon is conventionally used in multilayers mirrors for x-ray range [10].

The radiator candidates can be chosen from the elements in the periodic table with one of their absorption edges close to the desired frequency. This procedure is illustrated by Fig. 3. In Fig. 3 (which is based on data from Ref [15]), the photon energies corresponding to K, M, L absorption edges are plotted against the atomic number of elements which embrace almost the entire periodic table. One can see that for photon energy within the soft x-ray range (up to 2 keV), elements for the radiator can be chosen from K, L, or M branches. The further selection of one of these groups is based on the absorption factor as well as the dielectric constant factor \( |\varepsilon_1 - \varepsilon_2| \). At high photon energy (\( \geq 0.3 \text{ keV} \) or \( \lambda \leq 44 \text{ A} \)), the rule of thumb is to choose the heaviest elements as the radiator because their dielectric constants differ the most from the light spacer. To illustrate that, consider desired photon energy \( \approx 0.88 \text{ keV} \) corresponding to \( \lambda \approx 14 \text{ A} \). One can see from Fig. 3 that the radiator candidates are Ni and Ce (leaving aside Ne as a gas). Obviously, the best candidate is Ce.

The procedure described above enables one to select couples of elements each one consisting of a heavy element (radiator) + a light element (spacer) for generation of narrow line radiation with relatively high photon energy or short wavelengths (down to a few angstroms). At low photon energies, though, the light element may occur to be a good candidate as radiator. Consider an example when the desired photon energy of radiation is 100 ev which corresponds to \( \lambda \approx 110 \text{ A} \). One can see from Fig. 3 that the radiator candidates are Be, Al and Rb (with different shells). Since at low photon energy the absorption factor is larger for elements with high atomic number, Be is the best candidate. It is obvious, therefore, that in order to design a narrow line radiator for longer wavelengths (\( \approx 44 \text{ A} \) and longer) one has to use couples of two elements out of the "island of light elements", i.e., B, Be, and C. A combination of either two of these elements will form a system which would radiate at two frequencies out of three: 43.8A or 284.84 ev (C), 66.0A or 188.0 ev (B), and 111.04A or 111.0 ev (Be). Based on the results from previous sections, the energy of electron beam required for these structures to generate, are very low and can vary from 200-300 keV to 1 MeV.

![Fig. 4](image-url)  
Radiation efficiency \( \eta \) in number of photon / ev-electron-Mev versus photon energy \( h\omega \) in keV for the periodic structure with combinations of Be/Ce when electron beam energy \( E_e \) is 1 MeV and the spatial period \( l \) and total length \( L \) are optimized at the absorption edges of heavy elements. Inset: Actual peak of Ce

We consider some combinations of different media that constitute the periodic structure. In these examples, our choice of spacers will be Carbon (C). The advantages of C is its surface smoothness and stable boundary separation. The K absorption edges of C is at 0.283 keV. For the illustration sake, we choose Barium (Ba), Cerium (Ce), and Europium (Eu) as radiators (this choice being somewhat arbitrary). For practical applications, the choice of radiator depends on the fre-
quency range desired. The absorption edges of Ba, Ce, and Eu are at 0.78 kev (M edge), 0.883 kev (M edge), and 1.11 kev (M edge) respectively. These elements with high atomic number have the characteristic of large $f_1$ and $f_2$ compared to elements with low atomic number. The results of the previous sections indicate that in a soft x-ray range one should expect the resulting spectral density to have two peaks. The peak at low frequency is due to the element with low atomic number (which act then as a radiator) while the peak at higher frequency is due to the element with higher atomic number. As a demonstration, we use electron beam energy of 1 Mev which is readily available in university laboratories.

![Graph](Image)

Fig. 5. The same as Fig. 4 except Be/Ba as the materials constituting periodic structure. Inset: Actual peak of Ba

Figs. 4-6 shows the plots of radiation efficiency $\eta = E_\gamma^{-1} dN / d\omega$ versus photon energy for the cases with C as a spacer and Ba, Ce, or Eu as a radiator. The plots in Figs. 4-6 show the radiation from each structure optimized at the absorption edge frequency of each radiator. Note that we plotted the actual height and width of the peaks from heavy materials in the inset of Figs. 4-6 instead of drawing them in the main figures because these peaks are too high and too narrow to be depicted in the main figures. Indeed, the actual relative width of each peak is about $10^{-4}$ with radiation damping accounted for [see Eq. (15)], and its height is on the order of $10^{-8}$ which is about two order of magnitude larger than the scale of the main figure. It is obvious that the spectrum are maximum at the absorption edge frequency of each radiator. Since $|\Delta \varepsilon |^2$, which determines the intensity of radiation in direct proportion, decreases as $\omega$ increases, the peaks at high frequency have lower intensities than those peaks at low frequency. Based on the same reasoning, we expect the radiation at lower frequency generated by the light elements to be stronger than that of heavier elements emitted at high frequency. We also notice the abrupt decrease of radiation at frequency higher than the absorption edge frequency due to the discontinuous jump of absorption near the absorption edge.

The total length $L$ of the structure and its spatial period $l$ required for the maximum radiation for the C/Ba, C/Ce, and C/Eu structures when $E_\gamma = 1$ Mev are listed in Table 1. The value of parameters $L$ and $l$ for the structures in Tables 1 correspond to the structure used in Figs. 4-6, i.e. to the optimum structure at the absorption edge of the heavy elements. We observe from the tables that the period of the structure does not depend on the kind of materials whereas the total length of the structures varies a lot with different media. The variation in total length is due to the dependence of $L$ on absorption coefficient. The angle of emission which depends on the electron beam energy is approximately $17.7^\circ$ for the case of $E_\gamma = 1$ Mev.

Conclusions

We have investigated the role of photoabsorption and electron scattering in x-ray emission by electrons with low energies. We showed that energy of electron beam $E_\gamma$ from 100 - 200 kev to a few Mev is sufficient to give rise to the narrow line transition radiation (with a relative linewidth $\sim 10^{-4}$) in a multilayer structure with a short spatial period (50 A' to 2000 A'). We developed a procedure which allows us to choose appropriate
Table 1. The optimal spatial period $l_{opt}$ and optimal total length $L_{opt}$ for period structure with combination of Be/Ge, Be/Ce, Be/Ba, and Be/Eu for $E_0 = 1$ Mev around the radiation peak for higher frequencies corresponding to various transitions of different heavier elements (the M edges - for the Ba, Ce, and Eu).

<table>
<thead>
<tr>
<th>Radiator</th>
<th>$\lambda (\AA)$</th>
<th>$l_{opt} (\AA)$</th>
<th>$L_{opt} (\mu m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ce</td>
<td>14 (0.88 kev)</td>
<td>1297</td>
<td>5.97</td>
</tr>
<tr>
<td>Ba</td>
<td>18 (0.77 kev)</td>
<td>1396</td>
<td>9.07</td>
</tr>
<tr>
<td>Eu</td>
<td>11 (1.11 kev)</td>
<td>982</td>
<td>9.48</td>
</tr>
</tbody>
</table>

materials (both for radiator and spacer) in order to obtain radiation with narrow resonant peaks at the frequencies of absorption edges of various elements which are due to photo-ionization electron transitions from inner-atomic shells. We have developed a numerical procedure to optimize parameters of the periodic structure required to obtain maximum radiation. In this procedure, the spatial period, optimal total length, and resonant angle are evaluated. This completely specifies the design of the system.

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References


19. J. Ashkin and H. A. Bethe, "Passage of radiation through matter," in Experimental


