CHAPTER 4

Nonlinear Waves and Switching Effects at Nonlinear Interfaces

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1. Introduction

Over the past several years, a new direction in nonlinear optics has arisen, which can be described as self-action at nonlinear interfaces (i.e., an interface between linear and nonlinear transparent media). In the most studied configuration, light is incident from a linear medium (with susceptibility $\varepsilon_0$) onto a plane surface of nonlinear medium with a nonlinear refractive index (fig. 1). The studies of such an interface have followed an almost classical progression of alternating theoretical and experimental advances, which have resulted in an increasingly detailed understanding of the behavior, features, and limitations of nonlinear interfaces, and have brought into focus the remaining unsolved problems.

Studies of the behavior of nonlinear interfaces originated in theoretical analyses (Kaplan 1976, 1977, 1978, 1979, 1981a,b) which assumed that the incident and reflected fields (i.e., the fields in the linear medium) can be represented as infinite plane waves, and that the field in the nonlinear medium is either an infinite plane wave or a nonlinear evanescent wave, or some mix of both. These studies resulted in closed-form analytical expressions. We refer to them as the 'plane-wave theory'. A major result of the plane-wave theory was that under appropriate conditions the reflectivity of a nonlinear interface would exhibit a threshold behavior as a function of the intensity of the incident field: for intensities below the threshold value the interface would be totally reflecting, and for intensities above the threshold the interface would have a significant transmission. The plane-wave theory also predicted that for a range of input intensities just below the threshold value the reflectivity of the interface can be bistable.

The behavior predicted by the plane-wave theory suggested several applications (e.g., ultra-fast optical switches, scanners, and logic elements), and experiments were initiated to study these predicted phenomena. The initial experimental results were obtained using $\text{CS}_2$ as the nonlinear medium (with the so called positive or self-focusing nonlinearity, see below), and a high-power power.

Fig. 1. Wave diagram of the nonlinear interface. Ray trajectories in nonlinear medium: (1) traveling plane wave (PW), (2) longitudinally inhomogeneous traveling wave (LITW), (3) total internal reflection (TIR).
mode-locked ruby laser as the source (Smith et al. 1979, 1981, Smith and Tomlinson 1981a). The experimental results clearly displayed the predicted threshold behavior. They also seemed to show certain evidence of the predicted bistability (Kaplan 1976, 1977). However, because of the limited time resolution of the detection system, and the fact that the experiments were done with only a single input pulsewidth, this evidence was inconclusive.

The interpretation of the experimental results was also complicated by the fact that the experiments were done with a moderately tightly focused Gaussian beam (not an infinite plane wave). Various numerical simulations have been reported for a one-dimensional Gaussian input beam (i.e., a beam with a Gaussian distribution in the plane perpendicular to the interface, but which extends uniformly to infinity in the direction perpendicular to the plane of incidence) (Smith et al. 1981, Smith and Tomlinson 1981a, Rosanov 1979, Marcuse 1980, Kolokolov and Sukov 1978, Tomlinson et al. 1982), with contradictory results. Some of them (Rosanov 1979, and to some extent Marcuse 1980) endorsed the existence of bistability for the Gaussian-beam case, while others (Kolokolov and Sukov 1978) disputed it. Although some numerical simulations (Smith et al. 1981, Marcuse 1980, Tomlinson 1980) predicted the appearance of nonlinear surface waves, later results (Tomlinson et al. 1982, Kaplan 1981c), showed that these surface waves are prohibited under conditions implied in the works of Smith et al. (1981), Rosanov (1979), Marcuse (1980), and Kolokolov and Sukov (1978). The most recent modeling study confirmed the threshold behavior, but showed that the detailed behavior of the interface is quite complex (Tomlinson et al. 1982). The reflectivity exhibits not a single threshold, but a series of thresholds at increasingly higher input intensities. The transmitted light was shown to form self-trapped channels, which can be easily identified with the two-dimensional self-trapping (Askar'yan 1962, Talanov 1964, Chiao et al. 1964, Kelley 1965) well-known in the theory of the light propagation in a semi-infinite medium with positively nonlinear refractive index. In the case of the grazing incidence, though, these channels propagate along complicated curved paths, with each step in the reflectivity associated with the formation of an additional channel. The simulations used a steady-state model, which was not capable of proving, or disproving, the existence of bistability, but it was postulated that the nonlinear interface could not exhibit bistability, because of the lack of any memory mechanism to store the information on the previous field configuration (Tomlinson et al. 1982).

The next set of experiments made use of an 'artificial' nonlinear medium, with a very large, but also very slowly responding, effective nonlinearity (Smith and Tomlinson 1984). This made it possible to do experiments with a CW laser, and relatively detailed data were obtained on the reflectivity of a nonlinear interface as a function of the input-beam intensity. The results clearly displayed the multiple thresholds, and were in good agreement with the (one-dimensional) Gaussian-beam simulations. Bistability was not observed. However, it was
found that when the input intensity was lowered through a threshold value, the high transmission state would persist for the order of 300 times the response time of the nonlinearity. This quasi-bistability has not yet been accounted for theoretically. Hysteresis and, in fact, quasi-stability has also been recently reported (Altshuler et al. 1985) in an experiment on nonlinear reflection of picosecond pulses from a glass–liquid interface; Altshuler et al. (1985) attribute this effect to probable thermal changes in the near-surface layers in both glass and liquid.

The case of a negative (self-defocusing) nonlinearity has been much less thoroughly explored in experiment, but for this case, the plane-incident-wave theory predicts (Kaplan 1978, 1979, 1981a) an excitation of nonlinear waves of a new kind (described by Kaplan 1978, 1979, 1981a) as longitudinally inhomogeneous traveling waves which possess interesting physical features.

All of the nonlinear interface effects are caused by the competition between the mismatch in linear susceptibilities \( \Delta \varepsilon_L \) and the nonlinear component \( \Delta \varepsilon_{NL} \). If the linear difference is sufficiently small, this competition can result in a strong change of reflection even if the nonlinear component is small as well (as it usually is in optical range). As the intensity of the incident field increases, the penetrating (or evanescent) field in the nonlinear medium changes the index difference at the interface. At a critical intensity, an abrupt switch from total internal reflection (TIR) to partial transmission occurs if the nonlinearity is positive, or vice versa, if the nonlinearity is negative. In order for these effects to occur, it is necessary to have \( |\Delta \varepsilon_L/\varepsilon_0| \ll 1 \) (Kaplan 1976, 1977, 1978, 1979) (see below, section 2). The lack of these conditions leads to absence of the phenomenon. This may be the reason why such effects were not discovered earlier, either in theoretical or experimental research, e.g., in the work of Bloembergen and Pershan (1962), Bloembergen and Ducciung (1963), Bloembergen and Lee (1967) devoted to harmonic generation due to reflection of light from nonlinear dielectrics.

The optical hysteresis and bistable behavior suggested by the plane-wave theory (Kaplan 1976, 1977) can be viewed as phenomena quite typical for many nonlinear systems. The concept of bistability (multistability, in general) implies that the system can assume one of two (or more) stable stationary states under the same external conditions. Usually it correlates with hysteresis which occurs as some of the external parameters of the system (e.g., the intensity of incident radiation) vary. This phenomenon is well-known in physics, e.g., in the field of phase transitions, in the theory of nonlinear mechanical or electronic oscillators (Stoker 1950, Kaplan et al. 1966), etc.

One of the reasons that the predictions of bistable and hysteretic reflectivity at a nonlinear interface attracted attention is that in recent years optical bistability has become a very promising field in nonlinear optics (Gibbs 1985). Elements based on optical bistability can be utilized for optically controlled memory and logic operations in high-speed optical signal processing systems (Smith
and Tomlinson 1981b). Most known bistable optical devices comprise a Fabry–Pérot resonator filled with a nonlinear medium, as first proposed in Seidel (1969) and Szöke et al. (1969); bistability in these systems was first observed in Gibbs et al. (1976). In these systems, bistability is due to the presence of a resonator which provides a feedback. The media used might have a resonant saturated absorption (Seidel 1969, Szöke et al. 1969, Gibbs et al. 1976) or a nonresonant Kerr nonlinearity (Felber and Marburger 1976). The use of Fabry–Pérot resonators causes these devices to be strongly selective to the frequency of the incident light. In contrast to these devices, a nonlinear interface does not employ any resonators. Therefore, no resonant tuning of the laser frequency is required; moreover, a broad spectrum of input light can be used.

A number of new effects, which do not exist in resonators, have also been predicted for nonlinear interfaces, and have stimulated various proposals for applications. New optical switching devices related to the nonlinear interfaces have been proposed by Kaplan (1981b) and Smith and Tomlinson (1981a). It was suggested by Kaplan (1976, 1977) and studied in detail by Dlodlo (1981, 1983), to use nonlinear interfaces as bistable mirrors for self-pulsing lasers. Most recently, reversible optical computing circuits based on the plane-wave switching effect at nonlinear interfaces were proposed (Cuykendall and Andersen 1987). Interesting modifications of the nonlinear interface idea have appeared, including optical bistability in a frustrated-total-reflection optical cavity (Bosacchi and Narducci 1983), and in a dielectric cladded thin film near the TIR state (Khoo and Hou 1985). An enhancement of nonlinear interface effects by exploiting surface plasmons at metallic surface was proposed by Vysin et al. (1981) and experimentally explored by Martinot et al. (1985). Optical bistability using long-range surface plasmons was proposed (Hickernell and Sarid 1986), and surface resonance modes (attributed to dielectric spheres placed on a metal substrate) (Inoue 1987). An interesting analogy of bistable TIR at nonlinear interfaces was found (Slepyan 1985) for bistability of the reflection of light from metallic diffraction gratings (positioned at the nonlinear interface) in the vicinity of the Wood anomaly. One trend in the theory and applications of nonlinear interfaces is the study of nonlinear waveguides, e.g., dielectric waveguide formed by a linear dielectric layer sandwiched between two nonlinear layers or vice versa (see, e.g., Smith and Tomlinson 1981a, Stegeman 1982, Seaton et al. 1985, Vach et al. 1984 and references therein) which have promising characteristics.

Despite its deficiencies, the plane-wave theory of nonlinear interfaces has proven to be a useful tool, and a powerful stimulant to research in this field. The theory has suggested a number of interesting nonlinear effects:


(b) change and scanning of the refraction angle and reflection coefficient by varying the intensity of incidence light (Kaplan 1976, 1977, 1979);
(c) optically induced transparency of the interface by incident light with specific intensity (Kaplan 1976, 1977, 1979);

(d) change of penetration depth of the field into a reflecting medium in the regime of TIR (Kaplan 1977);

(e) excitation of nonlinear waves of a new kind (Kaplan 1977, 1978, 1979, 1981a,b) (longitudinally inhomogeneous traveling waves (LITW)) and effects related to LITW; namely

(f) strong nonlinear self-parallax of refracted rays along the interface (Kaplan 1979); and

(g) self-limitation of the energy flux of the refracted light (Kaplan 1979, Permyakov and Bagdasaryan 1978).

The theory has also been used to describe other situations, for example, bistability at an electro-optically driven interface (Kaplan 1981) which has recently been observed experimentally (Smith et al. 1982) and bistable reflection of light at the nonlinear optical waveguide junction (Kaplan 1981b). The main results of the plane-wave theory are described in section 2. The effects predicted by this theory in the particular cases of positive and negative nonlinearities are described in sections 3 and 4, respectively. The experimental and computer simulation results are discussed in section 5.

2. Plane-wave theoretical formalism

Let a plane wave with amplitude $E_{in}$ be incident from a linear medium with susceptibility $\varepsilon_0$ at the glancing angle $\psi$ (fig. 1) onto the boundary of a nonlinear medium whose susceptibility, $\varepsilon_{NL}$, depends on the field amplitude $E$ in the medium according to the equation:

$$\varepsilon_{NL}(E) = \varepsilon_0 + \Delta\varepsilon_L + \Delta\varepsilon_{NL}(|E|^2),$$

where

$$\Delta\varepsilon_{NL}(|E|^2) = \varepsilon_2 |E|^2,$$

$\varepsilon_2$ is the nonlinear coefficient (this is usually referred to as a Kerr nonlinearity), and $\Delta\varepsilon_L$ is the field-independent mismatch between susceptibilities of the two media. The most interesting self-action effects are expected when the nonlinear component $\Delta\varepsilon_{NL}$ (even if small) can compete with linear mismatch $\Delta\varepsilon_L$. Only then can the nonlinear component (caused by the field penetrating into the nonlinear medium) dramatically change the total difference $\Delta\varepsilon = \Delta\varepsilon_L + \Delta\varepsilon_{NL}$ between the susceptibilities of the two media. Such a change, in turn, can result in an abrupt jump of the reflectivity of the interface at some critical incident intensity, e.g., the switch from the TIR state into the transmission state. For the same reason, the incidence of the light has to be almost grazing, because the required magnitude of the glancing angle $\psi$ should be of the order of the critical
angle of TIR, which results in the above-mentioned conditions (Kaplan 1976, 1977),

$$\left| \frac{\Delta \varepsilon_1}{\varepsilon_0} \right| \sim \left| \frac{\Delta \varepsilon_{NL}}{\varepsilon_0} \right| \ll 1, \quad \psi \sim \left( \frac{\Delta \varepsilon}{\varepsilon_0} \right)^{1/2} \ll 1. \quad (3)$$

These conditions give $\psi \sim 1^\circ$ for $|\Delta \varepsilon_{NL}/\varepsilon_0| \sim 10^{-4}$. For most of the rest of this chapter we will assume that conditions (3) are valid. This will simplify the theory and make the phenomena independent of the polarization of the incident light (Kaplan 1976, 1977, 1979).

The field in the linear medium is represented in the form of two homogeneous plane waves, one of which is the incident wave and the other is the reflected wave, determined by an unknown complex reflection coefficient $r$. The wave equation for the complex amplitude of the field, $E$, in a nonlinear medium in the two-dimensional case can be written as

$$\frac{d^2 E}{dz^2} + k_0^2 E \left[ \frac{\varepsilon_{NL}(|E|^2)}{\varepsilon_0} - \cos^2 \psi \right] = 0, \quad (4)$$

where $k_0 = \omega \sqrt{\varepsilon_0}/c$, and the $z$-axis is taken to be perpendicular to the boundary. The total field is $\frac{1}{2} E \exp(-i\omega t) + c.c.$ By equalizing tangential components of the fields on both sides of the boundary, the generalized boundary condition for the refracted wave $E$ can be obtained (Kaplan 1977, 1978, 1979),

$$i \frac{dE(0)}{dz} + k_0 \sin \psi [2E_{in} - E(0)] = 0. \quad (5)$$

The expression for the reflection coefficient $r$ is

$$r = \frac{E(0)}{E_{in}} - 1, \quad (6)$$

which also follows from the continuity of tangential components of the fields at the interface. The boundary conditions are a very important part of any problem related to the nonlinear interface. The failure to define these conditions adequately could result in missing important effects. For example, inadequate conditions for the refracted plane wave in an earlier work (Boiko et al. 1975) made it impossible to determine the wave amplitude and refraction angle separately in the case of $\varepsilon_2 > 0$. This led Boiko et al. (1975) to the controversial conclusions regarding the existence of optical bistability even within plane-wave approach.

The field $E$ in the nonlinear medium is written in the form (Kaplan 1977, 1978, 1979, 1981a)

$$E = u(z) \exp \left[ ik_0 \int_0^z \xi(z) dz + i\phi + ik_0 x \cos \psi \right], \quad (7)$$

where $u(z)$ and $\xi(z)$ are real quantities, $u$ being the real amplitude of the field,
and $\zeta$ being the angle formed by the rays and the $x$-axis at a given point $z$; $\phi$ is a constant phase term. For plane waves (PW), $u$ and $\zeta$ are constant, and for surface (or evanescent) waves (SW), when TIR occurs, $\zeta = 0$. In general, $u$ and $\zeta$ are not constant.

In the theory of reflection from a semi-infinite medium, the conditions at infinity must be defined. Since there are no sources inside the nonlinear medium, only traveling waves should propagate away from the interface in the nonlinear medium (or SW, which do not carry any energy along the $z$-axis at all). Therefore, in eq. (7) for $z \to \infty$, the following conditions have to be satisfied (Kaplan 1976, 1977, 1978):

$$u \to \text{const} \equiv u_\infty \geq 0, \quad \zeta \to \text{const} \equiv \zeta_\infty \geq 0,$$

(8)

which is, essentially, the Sommerfeld radiation condition (the absence of the backward-traveling wave). Substituting the field in the form of eq. (7) into the wave equation [eq. (4)], we obtain its first integral,

$$I = \zeta u^2 = \text{const} \equiv \zeta_\infty u_\infty^2 \geq 0,$$

(9)

(which expresses the conservation of the energy flux) as well as the equation for the real amplitude $u$,

$$\frac{d^2 u}{dz^2} + k_0^2 u \left( \frac{\varepsilon_{NL}(u^2)}{\varepsilon_0} - \cos^2 \psi - \frac{l^2}{u^4} \right) = 0.$$

(10)

The first integral of this equation, which satisfies the radiation condition [eq. (8)], can be written in the form (Kaplan 1978):

$$\left( \frac{du}{dz} \right)^2 = k_0^2 \int u \left[ F(u) - F(u_\infty) \right] d \left( \frac{1}{u^2} \right),$$

(11)

where the nonlinear 'characteristic' function $F(u)$ is introduced,

$$F(u) = u^4 \left[ \frac{\varepsilon_{NL}(u^2)}{\varepsilon_0} - \cos^2 \psi \right].$$

(12)

Integrating eq. (11) and taking the boundary condition given by eq. (5) into account, one can obtain all possible wave solutions for the problem.

3. Positive nonlinearities

In the simplest case of cubic nonlinearity [eq. (2)] with $\varepsilon_2 > 0$ (i.e., Kerr nonlinearity), only two kinds of wave modes, with their intensities depending on $z$ only, can exist:

(a) the homogeneous plane wave (PW), $u(z) = \text{const}$, which corresponds to the transmission regime ($|r| < 1$); and
(b) the surface wave (SW), or evanescent wave, \( \xi = 0 \), which corresponds to TIR (\(|r| = 1\)).

Both of these waves are nonlinear analogs of corresponding linear waves, but differ now from those in their dependence on the incidence light intensity. (We should note that in two, or more, dimensions, a homogeneous plane wave is not a stable solution in a medium with a positive Kerr nonlinearity (Talanov 1964), and thus we would only expect the plane-wave theory to apply to the transmitted field in the region sufficiently close to the interface.) Using eqs. (4)–(7) with \( u \) and \( \xi \) constant, one readily obtains ‘Snell’s nonlinear formula’ for the transmission angle \( \xi \) (Kaplan 1976, 1977):

\[
\left(1 + \frac{\xi}{\psi}\right)^2 \left(\xi^2 - \psi^2 - \frac{\Delta \varepsilon_L}{\varepsilon_0}\right) = 4 \frac{\varepsilon_2}{\varepsilon_0} |E_{in}|^2,
\]

or ‘Fresnel’s nonlinear formula’ for the reflection coefficient \( r \),

\[
4 r \psi^2 + \frac{\Delta \varepsilon_L}{\varepsilon_0} (1 + r)^2 + \frac{\varepsilon_2}{\varepsilon_0} |E_{in}|^2 (1 + r)^4 = 0
\]

(see fig. 2). It was shown by Kaplan (1981a) that in the general case of arbitrary magnitude of nonlinearity \( \varepsilon_2 |E_{in}|^2 \) and the incident angle \( \psi \) [i.e., in the case when eq. (3) is not satisfied any more], the ‘Snell formula’ for the angle \( \psi_{NL} \) of a refracted plane wave is

\[
\left(\frac{\tan^2 \psi_{NL}}{\tan^2 \psi} - 1 - \frac{\Delta \varepsilon_L}{\varepsilon_0 \sin^2 \psi}\right) = 4 \frac{\varepsilon_2 |E_{in}|^2}{\varepsilon_0 \sin^2 \psi},
\]

(where now \( \tan \psi_{NL} = \xi/\cos \xi \)), while the ‘Fresnel formula’ for the amplitude \( u \) of reflected plane wave is

\[
4 \sin^2 \psi \left(1 - \frac{u}{|E_{in}|}\right) = \frac{u^2}{\varepsilon_0 |E_{in}|^2} (\Delta \varepsilon_L + \varepsilon_2 u^2).
\]

Fig. 2. Reflection coefficient \( r \) versus the incident light intensity \( |E_{in}|^2 \) at different glancing angles \( \psi \) in the case of negative linear mismatch of susceptibilities \( (\Delta \varepsilon_L < 0) \) and positive nonlinearity \( (\varepsilon_2 > 0) \).
Curves: (1) \( \psi > \psi_{cr} \); (2) \( \psi = \psi_{cr} \); (3, 4) \( \psi/\psi_{cr} > 1/2 \sqrt{2} \); (5) \( \psi = 2\sqrt{2} \psi_{cr} \) (nonlinear bleaching); (6, 7) \( \psi < 2\sqrt{2} \psi_{cr} \). (After Kaplan 1976.)
The formula for the reflection coefficient \( r \) follows from eq. (16) via eq. (6) which again yields eq. (14) where one has to replace \( \psi^2 \) by \( \sin^2 \psi \).

The nonlinear TIR state can be excited if linear mismatch is negative \( (\Delta \varepsilon_L < 0) \) and \( \psi < \psi_{cr} \), where \( \psi_{cr} = ((\Delta \varepsilon_L/\varepsilon_0)^{1/2} \) is a critical angle of linear TIR. The profile of wave intensity now is not exponential; it is obtained from eq. (11) with \( u_\infty = 0 \). In the case of Kerr nonlinearity [eq. (2)], the general equation [eq. (11)] under the condition of eq. (3), can then be written in the form

\[
\left( \frac{du}{dz} \right)^2 = k_0^2 u^2 (\psi_{cr}^2 - \psi^2 - \frac{1}{2} u^2).
\]

(17)

Integration of this equation yields (Kaplan 1976, 1977) the amplitude profile \( u(z) \) of a TIR evanescent field (surface wave) which has a ‘self-channel’ shape well-known in the soliton theory of two-dimensional self-trapping,

\[
u = \left( \frac{2\varepsilon_0}{\varepsilon_2} \right)^{1/2} \frac{\gamma}{\cosh(k_0\gamma z + C)},
\]

(18)

where \( \gamma = (\psi_{cr}^2 - \psi^2)^{1/2} \), and constant \( C \) is determined from the boundary conditions [eq. (5)], and can have from one to four values (for different \( \psi \) and \( E_{in} \)) which causes multistability and hysteretic jumps (see fig. 2). Indeed, from the complex boundary conditions [eq. (5)], one obtains a condition for the real amplitude \( u(0) \) of an evanescent field at the boundary:

\[
\left[ \frac{du(0)}{dz} \right]^2 + k_0^2 \psi^2 [u^2(0) - 4|E_{in}|^2] = 0,
\]

(19)

as well as a formula for the phases of the evanescent field \( \phi \) and reflected wave \( \phi_r \) [where \( \phi_r \) is determined by the relationship \( r = \exp(i\phi_r) \)],

\[
\phi = \frac{1}{2} \phi_r = \mp \arccos[2u(0)/E_{in}],
\]

(20)

where the upper sign corresponds to \( u(0) < 0 \) and the lower to \( u'(0) > 0 \). Now, eliminating \( du(0)/dz \) from eq. (19), and from eq. (17) (taken at \( z = 0 \)), one obtains an amplitude \( u(0) \) (Kaplan 1977),

\[
|u(0)|^2 = \frac{\varepsilon_0}{\varepsilon_2} \left[ \psi_{cr}^2 \pm \left( \psi_{cr}^4 - 8\psi^2 \frac{E_{in}}{\varepsilon_0} \right)^{1/2} \right].
\]

(21)

Stipulating that the amplitude \( u(0) \), eq. (21), and the phase \( \phi \), eq. (20), be real quantities, one obtains the conditions for the nonlinear TIR to occur,

\[
0 \leq u^2(0) \leq 4|E_{in}|^2, \quad 8\psi^2 \frac{E_{in}}{E_{in}} \frac{|E_{in}|^2}{|\psi_{cr}|} \geq \psi_{cr}^4 \varepsilon_0.
\]

(22)

Based on these conditions, one can show that the threshold grazing angle \( \psi_{th} \), at which the switch from nonlinear TIR to partial transmission occurs, is given by (Kaplan 1976, 1977)

\[
(\psi_{th}/\psi_{cr})^2 = 1 - 2\varepsilon_2 |E_{in}|^2/|\Delta \varepsilon_L|, \quad 4\varepsilon_2 |E_{in}|^2 < |\Delta \varepsilon_L| \leq \frac{1}{8}|\Delta \varepsilon_L|/\varepsilon_2 |E_{in}|^2, \quad 4\varepsilon_2 |E_{in}|^2 > |\Delta \varepsilon_L|.
\]

(23)
Therefore, for a fixed grazing angle $\psi$, the threshold input intensity $|E_{th1}|^2$, at which such a switch occurs, is given by

$$\frac{\varepsilon_2 |E_{th1}|^2}{\Delta \varepsilon_L} = \begin{cases} \frac{1}{2} (1 - \psi/\psi_{cr})^2, & \frac{1}{2} \sqrt{2} \leq \psi/\psi_{cr} < 1, \\ \frac{1}{8} \left( \frac{\psi_{cr}}{\psi} \right)^2, & 0 < \psi/\psi_{cr} \leq \frac{1}{2} \sqrt{2}. \end{cases} \quad (24)$$

Thus, if starting from TIR at the low-input intensity and gradually increasing the intensity, the interface switches from the TIR state ($|r| = 1$) to the partial-transmission state ($|r| \leq 1$, see fig. 2) at some intensity given by eq. (24). The reflection coefficient $r$ at this point is determined by eq. (14), where the value of $\varepsilon_2 |E_{th1}|^2$ is to be substituted from eq. (24). In the backward process (i.e., when the input intensity is decreased), the switch from the partial-transmission state to the TIR state occurs at the lower intensity, $|E_{th2}|^2$, which is determined by eq. (13) [or eq. (14)] by assuming $\xi = 0$ (or $r = 1$),

$$\frac{\varepsilon_2 |E_{th2}|^2}{\Delta \varepsilon_L} = \frac{1}{4} \left[ 1 - \left( \frac{\psi}{\psi_{cr}} \right)^2 \right], \quad (25)$$

i.e., $|E_{th2}|^2 < |E_{th1}|^2$ (see fig. 2). The jump of the reflection coefficient only occurs during the switch from TIR to transmission; when moving in the backward direction, there is only a jump in the phase of reflection.

One of the interesting features of nonlinear reflection is what can be called nonlinear bleaching (or optically induced transparency) (Kaplan 1976, 1977). It is seen from eq. (14) that at $\Delta \varepsilon_L < 0$, a ‘bleaching’ input intensity, $|E_{bl}|^2$ exists:

$$|E_{bl}|^2 \equiv |\Delta \varepsilon_L/\varepsilon_2|, \quad \text{i.e.,} \quad \Delta \varepsilon_L = -\Delta \varepsilon_{NL}, \quad (26)$$

at which there is no reflection at all ($r = 0$) for any glancing angle $\psi$ (see fig. 2). This phenomenon cannot be regarded as a nonlinear analog of the Brewster bleaching (since it does not depend on either the angle or polarization of incident light). The nonlinear bleaching is attributed to the fact that field equalizes the susceptibilities of the two media and, therefore, makes the boundary completely transparent. In this case, the jump from total reflection to total transmission occurs at $\psi = \psi_{cr}/2\sqrt{2}$ (see curve 5 in fig. 2). For CS$_2$, where $n_2 \sim 10^{-11}$ cgs esu, we have $E_{bl} \sim 1.8 \times 10^6$ V/cm at $\Delta \varepsilon_L \sim -10^{-3}$, in which case, $\psi = 0.54^\circ$.

4. Negative nonlinearities; longitudinally inhomogeneous traveling waves (LITW)

In a number of physical situations, the nonlinearity can be negative (i.e., $\varepsilon_2 < 0$), for instance, due to nonlinear resonant interactions (see, e.g., Butylkin et al. 1990), or due to giant nonlinearities, recently observed in some semiconductors (e.g., in InSb (Miller et al. 1980)). In such a case, the nonlinear analogs of linear
waves (i.e., PW and TIR) can exist as well. For plane waves (PW), the Snell [eqs (13), (15)], as well as the Fresnel [eqs (14), (16)], formulae remain valid (one has to take into consideration that now the sign of $\varepsilon_2$ is changed). In this regime it is also possible to vary the value of the transmission angle $\xi$ and reflection $r$ by changing the incident light intensity (fig. 3). Similarly to the 'positive' nonlinearity, a nonlinear total bleaching of the interface also exists; the 'bleaching' incident light intensity is given by the same expression, eq. (26), but now the value of the linear mismatch should be positive (fig. 3, curves 3 and 4).

In contrast to the 'positive' nonlinear case, the surface (evanescent) waves at 'negatively' nonlinear interface in the TIR regime exist in exactly the same range of linear mismatches and glancing angles [i.e., $\Delta\varepsilon_L < 0$ and $\psi < \psi_{cr} = |\Delta\varepsilon_L/\varepsilon_0|^{1/2}$] as in the linear case. However, now the amplitude profile of the SW, given by integration of eq. (11), is (Kaplan 1977)

$$u = \left[ \frac{2\varepsilon_0}{|\varepsilon_2|} \right]^{1/2} \frac{\gamma}{\sinh(k_0\gamma z + C)},$$

where $\gamma = (\psi_{cr}^2 - \psi^2)^{1/2}$, and $C$ is a constant determined by boundary conditions.

The amplitude of the field $u(0)$ at the boundary is given by eq. (21), where one has to take the negative sign in front of the radical. As $z \to \infty$, the evanescent field, eq. (27), decreases as $\sim \exp(-k_0\gamma z)$, just as in the linear case. But the depth of field penetration into the medium (at the half-intensity level), $L_{NL}$, which is determined by the shape, eq. (27), and the value of $u(0) \equiv u_0$, decreases (Kaplan 1977)

$$L_{NL} = L_0 \left[ 1 + \frac{2}{\ln 2} \ln \frac{1 + (1 + u_0^2/4\gamma^2)^{1/2}}{1 + (1 + u_0^2/2\gamma^2)^{1/2}} \right],$$

Fig. 3. Reflectivity $R = |r|^2$ as a function of the incident light intensity $|E_{in}|^2$, for a negative nonlinearity ($\varepsilon_2 < 0$), and $D = \Delta\varepsilon_L/\varepsilon_0$. Curves: (1) $D < 0$, $\psi^2 > |D|$; (2) $D = 0$; (3) $\psi^2 > \frac{1}{2}D > 0$; (4) $\psi^2 = \frac{1}{2}D$; (5) $\psi^2 < \frac{1}{2}D$; (6) $D < 0$, $\psi^2 < |D|$. (After Kaplan 1979.)
where $L_L = (\ln 2)/2k_0^2$ is the depth of the linear penetration. In a strong field, as $u_0^2 \gg \gamma^2$, one has $L_{NL} \approx 0.67L_L/u_0 \ll L_L$.

The most interesting feature in the case of negative nonlinearity is the feasibility of excitation (under some special conditions) of waves of a new kind, the so-called longitudinally inhomogeneous traveling waves (LITW), obtained analytically by Kaplan (1978, 1979, 1981a). Being inhomogeneous and non-planar near the boundary, these waves reduce to plane waves sufficiently far from the boundary. The concept of longitudinal inhomogeneity was introduced by Kaplan (1978) to distinguish this phenomenon from the self-actions due to transversely inhomogeneous waves, such as self-focusing (Askar’yan 1962, Talanov 1964, 1965, Chiao et al. 1964, Kelley 1965) and self-bending (Kaplan 1969) of the beams with limited cross-section.

Examination of eq. (11) shows that under the conditions

$$\frac{2}{3}q < \frac{|\epsilon_2|}{\epsilon_0} u_\infty^2 < q, \quad q \equiv \frac{\Delta \epsilon_L}{\epsilon_0} + \sin^2 \psi,$$

excitation of LITW is possible. Integration of eq. (11) yields two possible intensity profiles of LITW (Kaplan 1978, 1979):

$$u^2 = u_\infty^2 \pm 2B^2 \left\{ \frac{\sinh^{-2}}{\cosh^{-2}} \right\} (Bk_0^2 z + C), \quad B = \left[ \frac{3}{2} \frac{|\epsilon_2|}{\epsilon_0} u_\infty^2 - q \right]^{1/2}.$$

For the minimum possible energy density at infinity,

$$u_\infty^2 = u_m^2 = \frac{2}{3} \frac{\epsilon_0}{|\epsilon_2|} q,$$

only one (limiting) type of LITW remains:

$$u_{lim}^2 = \frac{2}{3} \frac{\epsilon_0}{|\epsilon_2|} \left[ \frac{1}{2} q + \frac{1}{(k_0^2 z + C)^2} \right].$$

The unique 'continuum' problem arises pertinent to the new solutions: we have only two conditions [which are equivalent to a single complex boundary condition eq. (5)] to determine three unknown constants [$u_\infty$ and $C$ in eq. (32), and $\phi$ in eq. (7)]. The radiation conditions, eq. (8), have already been used for the construction of LITW [eqs. (11) and (30)]. Therefore, there is a continuum of solutions even when boundary and radiation conditions are completely specified.

This situation differs drastically from the situation in a linear medium and a 'positive' nonlinear medium. An energy criterion for selecting a unique kind of physically realized LITW was proposed (Kaplan 1979) which is based on the minimization of wave energy density (simultaneously, it results in a maximum energy flux). The only 'surviving' kind of LITW occurs to be a limiting type, eq. (32), in which case $u_\infty^2$ reduces to the minimum possible value for LITW.
equal to $u_0^2$ [eq. (31)]. This choice was verified analytically (Kaplan 1979) by an asymptotic method whereby a weak absorption is introduced and the asymptotic results are obtained when the absorption tends to zero. A small parameter of this limiting transition occurred to be $(k_0 L_{ab})^{-2/5}$, where $L_{ab}$ is a characteristic length of the wave absorption.

Now, when the unique 'surviving' type of LITW (but not unique solution yet), is found, one can obtain all the characteristics of these waves, in particular, their amplitude at the boundary, $u_0$ (Kaplan 1979, 1981a). Based on this result and using the relationship for $u_0$ for PW, eq. (16), it is easy to find the domains of existence of all of the possible waves (i.e., PW, LITW and TIR) in the system (Kaplan 1981a). These domains are shown in fig. 4 in the space of generalized parameters $P$ and $Q$,

$$
P = \frac{\varepsilon_2 |E_{in}|^2}{\varepsilon_0 \sin^2 \psi}, \quad Q = \frac{\Delta \varepsilon_L}{\varepsilon_0 \sin^2 \psi} + 1. \tag{33}
$$

The LITW excitation gives rise to new effects. First of all, as in the 'positive' nonlinear case, it leads to the hysteretic behavior of a system under definite conditions, but now the hysteresis jump occurs between states of two different traveling waves, PW and LITW (in contrast to 'positive' nonlinearity, where they occur between PW and TIR). Hysteresis occurs only under the conditions (Kaplan 1979, 1981a)

$$|\varepsilon_2| |E_{in}|^2 > \Delta \varepsilon_L > 2\varepsilon_0 \sin^2 \psi.$$

Fig. 4. Diagram of wave states in the space of parameters $P$ and $Q$. Curve (1) is the boundary between PW and LITW states; curve (2) is the boundary of transition jump PW $\rightarrow$ LITW; curve (3) is the boundary of reverse transition jump LITW $\rightarrow$ PW. The space between curves (2) and (3) is the hysteresis region. (After Kaplan 1981a.)
The behavior of reflectivity for different parameters of a system is shown in fig. 3 (in particular, one can see hysteresis, curve 5, and self-bleaching, curves 3 and 4). In this figure, $E^2_{cr}$ is the critical intensity of incident light required for excitation of LITW for each particular case, and $D \equiv \Delta \varepsilon_1/\varepsilon_0$.

Another effect which is of interest for applications, consists of the self-limitation of the energy flux of LITW penetrating into a non-linear medium (Kaplan 1979, Permyakov and Bagdasaryan 1978). This is a direct consequence of the selection of a unique value $u_{\infty} = u_m$ [eq. (31)]. Behavior of the energy flux $I$ is shown in fig. 5 for hysteretic and non-hysteretic situations.

One of the most interesting effects attributed to the excitation of LITW is self-paralllex (Kaplan 1979), i.e., displacement of the refracted rays along the interface. This effect is not a nonlinear analog of Goos–Hänchen displacement of Gaussian beams under TIR. This is because, in contrast to Goos–Hänchen, self-parallax is now valid for plane waves and for the transmission regime, rather than for TIR. It corresponds to the initial ‘sticking’ of the rays to the interface (fig. 1, curve 2) which occurs because of conservation of the energy flux [eq. (9): the angle $\xi$ should increase as the intensity $u^2$ decreases], and is not related to the existence of hysteresis. The ray trajectory in space is determined by (Kaplan 1979)

$$x = z(3/Q)^{1/2} + l(1 + z_l/z)^{-1},$$

where $l$ is the parallax (ray displacement) for $z \to \infty$ and $z_l$ is the characteristic depth for half of the parallax; $l$ and $z_l$ are determined by $E_{in}$, $\psi$ and $\Delta \varepsilon_1$. Let $k_0 = 10^5$ cm$^{-1}$, $\Delta \varepsilon_1/\varepsilon_0 = -10^{-4}$, $\psi \sim 0.7^\circ$ and $|\varepsilon_2/\varepsilon_0| = 10^{-10}$ cgs esu; then the critical value of $E_{in}$ for excitation of LITW is $E_{cr} = 1.15 \times 10^5$ V/cm, and if $E_{in} = 2E_{cr}$, one obtains $l = 1.2$ cm and $z_l = 0.01$ mm; the parallax is very strong and occurs at a very small depth.

The existence of LITW in a system with ‘negative’ nonlinearity and its absence in a ‘positive’ nonlinear system poses a number of questions related to the kind of nonlinearity of the medium, which may cause LITW, such as: is it possible to predict what kinds of nonlinearities and system characteristics give rise to

![Fig. 5. Energy flux $I$ of transmitted waves as a function of the incident light intensity $|E_{in}|^2$ for hysteretic and nonhysteretic situations. (After Kaplan 1979.)](image-url)
LITW and what kinds of nonlinearities do not allow LITW? What are the parameters of possible LITW if allowed?

These questions have been addressed by a theorem formulated and proved by Kaplan (1978, 1979). This theorem relates the existence of different kinds of traveling waves to the behavior of a 'characteristic' nonlinear function \( F(u) \), eq. (12) above. The main conclusion of this theorem is that the existence of LITW in a transparent medium is possible if, and only if:

(a) there is a range \( V \) in which \( F(u) > 0, u \in V \);

(b) \( F(u) \) falls at least somewhere in \( V \) (i.e., there is at least one interval \( W \subset V \) where \( F(u) \) falls monotonically);

(c) if these conditions are satisfied, the LITW, if excited, can have a value of \( u_{\infty} \) which must belong only to this falling interval, i.e., always \( u_{\infty} \in W \).

The proof of this theorem is based on an investigation of the behavior of the integral in the right-hand side of eq. (11).

One of the main implications of this theorem is that LITW can be excited only in a nonlinear medium whose nonlinearity \( \varepsilon_{\text{NL}}(u^2) \) has at least one interval of fall. On the other hand, LITW are prohibited in the medium with increasing function \( \varepsilon_{\text{NL}}(u^2) \). This is the reason why excitation of LITW is prohibited for 'positive' nonlinearity.

The 'continuum problem' arises for all kinds of LITW. Therefore, the special issue in the theory of two-dimensional nonlinear waves is related to 'survival' of nonlinear transmitted waves in the case of arbitrary nonlinearity. Several theorems related to this problem were formulated by Kaplan (1979). In particular, a complete class of functions \( F(u) \) were found which allow for the 'surviving' LITW, and it was proved that the principle of minimization of wave energy remains valid throughout this class.

5. Experiments

For the purpose of describing the experiments, it is convenient to use the refractive index, \( n \), rather than the susceptibility, \( \varepsilon \). We write the index of the nonlinear medium as

\[
n(I) = n_0 + \Delta + n_2 I
\]

where \( n_0 \) is the index of the linear medium, \( \Delta \) is the mismatch between the refractive indices of the two media, which does not depend on the light intensity, \( I \), and \( n_2 \) is the nonlinear index. In terms of the parameters used earlier in this paper, we can write (for \( \Delta \ll 1; n_2 I \ll 1 \))

\[
\Delta = \Delta \varepsilon L / 2n_0, \quad n_2 I = \varepsilon_2 |E|^2 / 2n_0.
\]

The first experiment to observe the effects suggested by theory (Kaplan 1976,
1977) was reported by Smith et al. (1979), and Smith and Tomlinson (1981a). In this experiment, the liquid CS$_2$ was utilized as a nonlinear medium with positive nonlinearity ($\varepsilon_2 > 0$), and glass was used as a linear medium. The temperature regulation of the cell served as a fine control of the difference $\Delta\varepsilon_L$ between both susceptibilities required by relationship, eq. (3). The experiment is briefly described below following (Smith et al. 1979, 1981, Smith and Tomlinson 1981a).

Experiments were performed with the apparatus shown in fig. 6. The input pulse was generated by a mode-locked traveling-wave ruby laser. Internal Fabry–Pérot mode selection was employed so that the laser output consisted of a train of $\approx 1$ ns pulses. The absence of internal structure in these pulses was verified with a streak camera. A single pulse was selected from this train and directed onto the setup shown in fig. 6. A portion of the beam was sampled and directed with an optical delay of $\approx 6$ ns onto a fast photodiode. The remainder of the beam was focused into the glass cell containing CS$_2$. The glass was chosen to have an index of refraction close to that of CS$_2$ at room temperature in order to satisfy the conditions of eq. (3). The entire cell was placed in a temperature-controlled holder and the temperature was adjusted so that for the 694.3 nm wavelength of the ruby laser, the index difference $\Delta \approx 10^{-3}$. This corresponded to $\psi_{cr} \approx 2.0^\circ$ in the glass cell.

The reflected beam was monitored by the same photodiode used to monitor the incident pulse, and the diode output was displayed on a fast oscilloscope. The detector–oscilloscope combination had a measured response time of 320 ps. Figure 7a shows the measured incident and reflected pulse shapes for an incident intensity slightly above the threshold for hysteresis. The dashed curve is from calculations, based on the plane-wave theory (Kaplan 1976, 1977) [see eq. (14)], and was fitted to the data by adjusting the ratio of the peak intensity to the threshold intensity $I_0 = |E_{th1}|^2$, which corresponds to the switch from TIR to partial transmission. The fit becomes less good as the peak input intensity is increased (see fig. 8), however, the plane-wave theory (Kaplan 1977) still

![Fig. 6. Experimental set-up. (After Smith et al. 1979.)](image-url)
Nonlinear waves and switching effects at nonlinear interfaces

![Graphs showing intensity and reflection coefficient vs. incident intensity](image)

Fig. 7. Experimental measurements of incident and reflected pulse shapes for \( \psi/\psi_e = \frac{1}{3}\sqrt{2} \). (a) The solid curve is the incident pulse, and the dots are the experimental measurements of the reflected pulse. The dashed line is the reflected pulse calculated from an incoherent plane-wave analysis for \( I/I_0 = 1.125 \) (see text). (b) The experimental data in (a) plotted in a way that demonstrates the optical hysteresis observed. (After Smith et al. 1979.)

...accurately predicts the threshold intensity \( I_0 = |E_{th1}|^2 \) [eq. (24) above] for the onset of the switch from TIR to transmission. Moreover, one can see from figs. 7 and 8, that always (Kaplan et al. 1982)

\[
I_2 \approx \frac{1}{2} I_0,
\]

which is also consistent with the plane-wave theory prediction [see eq. (25) as compared with eq. (24)].

From the published value \( n_2 = 3 \times 10^{-8} \) (MW/cm²)⁻¹ for CS₂ (Moran et al. 1975), the theoretical value of \( I_0 = 8.1 \times 10^9 \) W/cm² was computed based on eq. (24) above. The experimentally measured value was \( I_0 = 7.5 \times 10^9 \) W/cm², in very good agreement with the theoretical value. In fig. 7b, the same experimental data as in fig. 7a are shown but replotted in the form of reflectivity versus input intensity. This form emphasizes the optical hysteresis observed. However, because of the limited time resolution of the detection system, and the...
Fig. 8. Experimental measurements of incident and reflected pulse shapes for $\psi/\psi_{cr} = \frac{1}{\sqrt{2}}$. The solid curves are the incident pulse and the dots are the experimental measurements of the reflected pulses. The dashed lines are from the incoherent plane-wave analysis fitted to all four curves with a single value of the adjustable parameter, $I_0$. (After Smith et al. 1981.)

fact that the experiments were only done for a single input pulsewidth, the observed hysteresis does not prove that the reflectivity is bistable.

Finally, the measured values of threshold intensity $I_0$ as a function of angle of incidence $\psi$ are shown in fig. 9. We see that the results are again in good agreement with the predictions of the plane-wave theory (Kaplan 1977) [see eq. (24) above].

Some of the experimental results show substantial disagreement with plane-wave calculations. For instance, for sufficiently large peak input intensities, the measured reflected pulse shapes deviate significantly from those obtained from calculations. It was believed and seemingly backed by computer simulations by Smith et al. (1981), Smith and Tomlinson (1981a), Marcuse (1980) that at least some of these disagreements could be caused by the excitation of a nonlinear surface wave propagating along the interface with no decay, beginning from the point of reflection of the Gaussian beam. Indeed, it was shown by Tomlinson (1980) that a surface nonlinear wave in the form of a stationary channel can propagate along the interface, traveling from $x = -\infty$ to $+\infty$, in the absence of any wave incident from either linear or nonlinear media. [Similar waves were discussed previously (Litvak and Mironov 1968, Alankyan 1967) in application to plasmas]. However, using a general theory of the stability of solitons of a cubic-nonlinear Schrödinger equation (Zakharov and Shabat 1971), it was pointed out by Kaplan (1981c) that if the surface nonlinear wave (in fact, a soliton solution) propagates in the direction $x \to \infty$ beyond the point where the
incident Gaussian beam hits the interface, then the same soliton-like surface wave (albeit with a spatially shifted maximum) must exist even before this point, i.e., this wave can be observed even when $x \to -\infty$. This would be inconsistent with the formulation of the problem (indeed, it is presumed that at $x \to -\infty$, the entire energy of the field is concentrated in the linear semi-space); therefore the 'semi-infinite' nonlinear surface wave (Smith and Tomlinson 1981a, Marcuse 1980) is prohibited.

The computer simulation (Tomlinson et al. 1982) confirmed this result. Furthermore, it showed that when the intensity of a laser Gaussian beam exceeds a critical level, the TIR state is drastically disturbed. The laser beam instead of being totally reflected, breaks through the interface and propagates in the nonlinear material away from the interface. The intensity of the beam at this moment is sufficiently high for the beam to form a self-trapped channel, as shown in fig. 10. If the incident intensity increases further, a second channel is formed (Tomlinson et al. 1982). Similar multisoliton emission from a nonlinear waveguide has recently been demonstrated in a computer simulation by Gubbels et al. (1987). It is natural to expect formation of more and more channels as the intensity increases, which would correspond to the multisoliton solution (Zakharov and Shabat 1971). The formation of each self-trapped channel is accompanied by a downward jump in the reflection coefficient (Tomlinson et al. 1982). Since the self-trapped channels form right at the interface, resulting in a strong spatial modulation of the intensity at the interface, it is clear that this phenomenon cannot be described by the plane-wave theory, in which it is assumed that the transmitted field is a homogeneous plane wave. The simula-
Fig. 10. Perspective plot showing the results of numerical simulations of the behavior of a nonlinear interface for an incident one-dimensional Gaussian beam. This plot is for an input intensity such that a self-focused channel forms in the nonlinear medium. For this figure (from Tomlinson et al. 1982), the definitions of the x- and z-coordinates are interchanged from those used in the rest of this paper.

...tions used a steady-state model, which is not capable of proving, or disproving, the existence of bistability, but it was postulated that a nonlinear interface cannot exhibit bistability, because of the lack of any memory mechanism to store the information on the previous field configuration. (This argument does not exclude bistable behavior for the case of other nonlinearities, such as either thermal or diffusion nonlinearities, that are either non-instantaneous and/or non-local.)

In later experimental research (Smith and Tomlinson 1984), an 'artificial' nonlinear medium (dielectric spheres suspended in a liquid) was used in order to observe switching at a nonlinear interface in the cw regime. The results of these observations were very close to the computer results (Tomlinson et al. 1982). Figure 11 shows the results of the computer calculations for a one-dimensional Gaussian beam. The experimental reflectivity data shown in fig. 12 are in good qualitative agreement with this model. Perhaps more striking is the excellent agreement between the experimental data for the critical intensity for reflectivity 'jumps' and the predictions of the Gaussian-beam model (fig. 13).
For short observation times, hysteresis was observed. Only the upper branch of the reflection hysteresis characteristic was stable, however. The lower branch would decay to the upper branch with a surprisingly long time-constant of $\sim 10^3 \times$ (the response time of the nonlinear medium). The nature of this effect
remains unclear, although it suggests, at least, that for any application with a pulse mode of operation, the nonlinear interface will demonstrate hysteresis. Indeed, the more recent experimental work (Delfino and Mormile 1985) done with essentially the same kind of media as that of Smith and Tomlinson (1984) using the long-pulse mode of operation, revealed very distinct hysteretic jumps. Furthermore, another more recent experiment (De Nicola et al. 1989) showed that under certain conditions the lower branch of the hysteretic curves can also be stable (see note added in proof). Delfino and Mormile (1985) have also experimentally observed the large nonlinear Goos–Hänchen effect suggested by Kaplan (1977) and obtained in a computer simulation (Tomlinson et al. 1982). The Goos–Hänchen effect consists in the displacement of a reflected beam along the interface with regard to the point of incidence (see, e.g., Litsch 1970, 1971, 1972). It was shown by Tomlinson et al. (1982) that for the intensities of the incident beam slightly lower than the critical intensity of formation of the first self-trapped channel, the point of reflection at the nonlinear interface is moving very far away from the point of incidence as the incident intensity increases.

6. Conclusions

In this chapter, we have attempted to review the present status of the theoretical analysis of nonlinear interface phenomena, and to present some of the available
experimental data. In general, the experimental results are in good agreement with theoretical predictions. The major area where uncertainty remains is the question of optical bistability for a fast Kerr-nonlinear material and a single nonlinear interface, with the local nonlinearity. No clear evidence from either theory or experiment exists for the presence of two completely stable states when finite-size beams are involved.

There are many phenomena predicted of the theoretical analyses that have not yet been observed experimentally. This is primarily due to the difficulty of finding suitable nonlinear media. Recent studies of nonlinear coefficients in highly transparent optical glasses (Friberg and Smith 1990) may lead to all-glass interface structures that will allow detailed experimental verification of many of the novel phenomena predicted by the analysis presented in this paper.

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Note added in proof

Since the original draft of this chapter was submitted, quite a few publications on the subject have appeared. Space does not allow us to review them in detail or even make a complete listing of them; we will briefly survey only those recent publications that are closely related to the issue discussed here.

Since the very existence of CW hysteresis and optical bistability, as well as a nonlinear surface wave in the situation when the incident beam has limited size, remained in question, numerous studies have been undertaken to clarify this situation. Computer simulation (Andersen 1988) showed that if the EM wave interaction with a Kerr-nonlinear material at a nonlinear interface is diffusive (or nonlocal), a nonlinear surface wave can couple to an incident Gaussian beam, a situation prohibited for a regular Kerr-nonlinear interface (Kaplan 1981c, Tomlinson et al. 1982). In an experiment (De Nicola et al. 1989) on the reflection of laser beam from the interface between a linear (glass) and an “artificial” nonlinear material (i.e. dielectric spheres suspended in liquid) similar to that of Smith and Tomlinson (1984), the reflection versus incident intensity curve exhibited hysteresis, and the lower branch of the hysteresis loop could be very stable and robust for optimized conditions. This indicated that a real CW hysteresis (and, therefore, optical bistability) can exist in the system, and it was attributed by the authors to the nonlocal property of interaction. In another experiment (Strobi and Cuykendall 1989, 1990), switching at a nonlinear interface (also with an “artificial” nonlinear material) was observed with enhanced contrast, and this was attributed by the authors to the saturation of the nonline-
arity, which results in suppression of the multiple self-trapped channels and associated multiple thresholds observed earlier (Smith and Tomlinson 1984), see figs. 11 and 12 here. An apparently related effect of stable (i.e. not broken into self-trapped channels) propagation of the beam refracted from a nonlinear interface between glass and a Kerr-nonlinear liquid has been reported by Li et al. (1990). Self-induced reflection (Mohebi et al. 1989) as well as energy limiting (Mohebi et al. 1990) have also been experimentally observed at a nonlinear interface between a dye solution and fused silica, using the large resonant nonlinearity of the saturably-absorbing dye solution.

Nonlinear interfaces with liquid-crystal layers have attracted much interest recently, apparently because of relative experimental simplicity: CW lasers with moderate power and almost any wavelength usually suffice as source of radiation. In most cases the originally expected nonlinearity due to reorientation of molecules in the optical field did not produce any significant contribution; the observed optical bistability was basically due to a thermal effect. Analytical as well as experimental investigations (Peschel et al. 1988, Boiko et al. 1988, Khoo et al. 1989, Haelterman and Waelbroek 1990) of optical transmission (and reflection) through a nonlinear layer, in particular a liquid-crystal layer sandwiched by glass prisms in a configuration similar to that of Khoo and Hou (1985), confirmed the existence of large hysteresises and optical bistability (with the nonlinearity apparently attributed to thermal effects); computer simulation of its dynamics was done by Khoo and Zhou (1989). Bistability due to surface plasmon-polaritons at a liquid-crystal/glass interface has been observed and interpreted by Martinot et al. (1985) and Innes and Sambles (1989). Arakelian et al. (1986) experimentally observed bistable excitation of a nonlinear surface waves in a prism-metal liquid-crystal film using a coupling technique originally described by Chen and Carter (1982, 1983). Although CW bistable effects in liquid crystals are unusually well pronounced, they seem to have marginal application potential, aside from demonstration purposes, because of the very slow switching time of the thermal nonlinearity.

Some new configurations related to nonlinear interfaces have been proposed and investigated. Theoretical work by Voronko et al. (1986) discussed a multistable excitation of a channeled surface polariton in a nonlinear resonator with mirrors reflecting the surface wave back on itself. Bistability and redirecting of optical beams has been observed near Brewster's angle by Schützgen et al. (1988) using a nonlinear ZnSe prism. The possibility of bistable effects using normal incidence at a nonlinear dispersive interface has been discussed by Cao and Banerjee (1989) and Banerjee et al. (1990). The theoretical and computer simulation aspects of the physics of nonlinear interfaces are of significant interest, especially for the realistic configurations. Extensive computer simulations of a three-dimensional Gaussian beam incidence on a nonlinear interface (Andersen and Regan 1989) revealed behavior patterns some of which differ qualitatively from those for the two-dimensional simulation (Tomlinson et al. 1982). How-
ever, for Kerr-nonlinear material these simulations verified the absence of a nonlinear surface wave coupled to the incident beam. Some interesting results were obtained in computer simulation (Aceves et al. 1989) of reflection of soliton-like light beams at nonlinear–nonlinear interfaces, although these results cannot be extended to the linear–nonlinear interfaces. An analytical theory of radiative losses of solitons in these systems has been developed by Kivshar et al. (1990).

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