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## SHORT WAVELENGTH COHERENT RADIATION: GENERATION AND APPLICATIONS

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## Low Relativistic Electron Beams Traversing a Superlattice as a X-ray Source

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### Abstract

Soft X-ray can be generated by low relativistic electron beams traversing through a medium with short spatial period. The amount of radiation generated is reduced owing to photon absorption and electron scattering. We investigate the influence of photon absorption and electron scattering on the amount of radiation, the critical electron energy and the total length of the superlattice. The results indicate that a heavy medium may be a better radiator for our system.

### Introduction

When fast electrons pass through a periodic medium, electromagnetic wave is generated (resonant transition radiation)<sup>1-3</sup>. The resonant angle  $\vartheta_n$  with respect to the electron beam and wavelength  $\lambda$  for this radiation is related by so called resonant condition

$$\sqrt{\bar{\epsilon}} \cos \vartheta_n = c/v - n\lambda/l \quad (1)$$

where  $\bar{\epsilon}$  is the average dielectric constant,  $v$  is the velocity of electron,  $n$  is an integer and  $l$  is the spatial period of interfaces. This is a condition of constructive interference of electromagnetic field at a distance point. Usually  $l \gg \lambda$ , so that ultra-relativistic electron are required to satisfy Eqn (1) and to emit radiation. With the progress in technology, we can construct superlattice with short spatial period  $l$ . It was shown previously that a system composed of thin layers ( $l \leq 500 \text{ \AA}$ ) enables one to reduce the critical energy for satisfying Eqn (1) to a few tens Kev or lower<sup>4,5</sup>. The fortunate factor is that the layer structures with  $l < 100 \text{ \AA}$  and with broad spectrum of materials are well known and are widely explored as X-ray mirrors for the same range of frequency<sup>6,7</sup>. In the proposal<sup>4,5</sup>, the electron beam is supposed to pass through such a solid-state structure. However, photon absorption and electron scattering prevent one from using too low electron energy. It turns out that a slightly increase of the energy of electron beam overcomes this problem while this energy is still low. In this paper, taking photon absorption and scattering electrons into consideration, we obtain the optimal range of electron energy  $E_0$  (which appears to be between a few hundred Kev and a few Mev), the total thickness  $L$  of the superlattice and maximal radiation for a given frequency of radiation  $\omega$ .

In this paper, we show that usually the effect of photon absorption is dominant over electron scattering. Indeed, the photon absorption

length<sup>8</sup> defined as  $1/\bar{\mu}$  is of the order of micron at a wavelength about  $10 \text{ \AA}$  where  $\bar{\mu}$  is the average photon absorption coefficient. On the other hand, the critical length for scattering electron<sup>9</sup> defined as  $E_0(dE/dx)^{-1}$  (where  $dE/dx$  is average energy loss per unit path length) is usually of the order of millimeter for electron energy  $E_0$  of 1 Mev. Permittivity of the material also strongly affect the optimal radiation and the shape of the radiation spectrum. Usually<sup>10,11,13</sup>, a light medium with small atomic number is used as a radiator and air is employed as a spacer in a system utilizing ultra-relativistic electrons. However, a heavy medium with atomic number may be an appropriate candidate as a radiator (and a light medium as a spacer) for producing strong radiation with narrow spectral width in our system since we have to use a solid-state structure.

### Emission in a Periodic Absorbing Media

The differential cross section for transition radiation including photon absorption in a multilayer system can be expressed in the following form<sup>11,12,13</sup>

$$\frac{d^2N}{d\Omega d\omega} = F_1 F_2 F_3 \quad (2)$$

where  $F_1$  is the differential cross section for one single interface,  $F_2$  denotes the coherence interference of radiation in a single plate and  $F_3$  represents the coherence summation of radiation from each layer (definition of  $F_1$ ,  $F_2$  and  $F_3$  is given below). The unit of the differential cross section is number of photon per unit solid angle in steradian per unit photon energy in ev per electron. One of the characteristics of transition radiation is the requirement of minimum distance for significant generation of radiation. The distance is known as formation length<sup>3,13</sup>

$$Z_i = \pi\lambda(1/\beta - \sqrt{\epsilon_i - \sin^2\vartheta})^{-1} \quad (3)$$

where  $Z_i$  ( $i=1,2$ ) are formation length of the two media,  $\epsilon_i$  ( $i=1,2$ ) are the permittivity of the two media,  $\beta=v/c$ ,  $v$  is the speed of the electron,  $c$  is the speed of light and  $\vartheta$  is the angle of emission with respect to the path of electron. Usually<sup>2,3,10,11,13</sup> the permittivity is calculated with the plasma frequencies  $\omega_i$  ( $i=1,2$ ) for the two media

$$\epsilon_i = 1 - (\omega_i/\omega)^2 \quad (4)$$

where plasma frequencies  $\omega_i$  ( $i=1,2$ ) =  $2c\sqrt{\pi\tau_0 N_i Z_{\alpha_i}}$ , classical electron radius  $\tau_0 = 2.82 \times 10^{-13}$  cm,  $N_i$  ( $i=1,2$ ) are the number of atom per  $\text{cm}^3$  in each medium and  $Z_{\alpha_i}$  ( $i=1,2$ ) are atomic numbers (for sufficiently high frequency). However,  $\omega_i$  cannot be used in Eqn (4) for most of the material in the soft X-ray photon energy (0.1 - 2 Kev)<sup>8</sup>. Instead of plasma frequency  $\omega_i$  in Eqn (4), we must use parameters  $\omega_{\alpha_i}$  with tabulated data<sup>8</sup> to calculate permittivity where  $\omega_{\alpha_i}$  ( $i=1,2$ ) =  $2c\sqrt{\pi\tau_0 N_i f_{R_i}}$  and  $f_{R_i}$  ( $i=1,2$ ) are the real part of atomic scattering factors  $f_i$ <sup>8</sup>. This modification affects the spectral shape. This point will be emphasized later.

The differential cross section for single interface is given by <sup>3</sup>

$$F_1 = \frac{\alpha(\Delta\varepsilon\beta)^2}{\pi^2\omega} |G(\beta, \varepsilon_1, \varepsilon_2, \vartheta)|^2 \quad (5)$$

where  $\Delta\varepsilon = \varepsilon_1 - \varepsilon_2$ , fine structure constant  $\alpha = 1/137$  and

$$G(\beta, \varepsilon_1, \varepsilon_2, \vartheta) = \frac{\sin\vartheta \cos\vartheta (1 - \beta^2 \varepsilon_2 - \beta \sqrt{\varepsilon_1 - \varepsilon_2} \sin^2\vartheta)}{(1 - \beta \varepsilon_2^2 \cos^2\vartheta)(1 - \beta \sqrt{\varepsilon_1 - \varepsilon_2} \sin^2\vartheta)(\varepsilon_1 \cos\vartheta + \sqrt{\varepsilon_1 \varepsilon_2 - \varepsilon_1^2} \sin^2\vartheta)}$$

According to the above, formula the differential cross section  $F_1$  is proportional to  $(\beta\Delta\varepsilon)^2$  and depends on angle of emission in a complicated fashion. A simple picture of transition radiation ensues when speed of electron and frequency of radiation are low so that one of the media behaves like metal. Under these conditions, we have dipole radiation pattern. As electron energy increase, the radiation concentrates in a narrow cone.

$F_2$  accounting for coherent summation of radiation in two neighboring interfaces assumes the interference pattern of two sources

$$F_2 = 4\sin^2(l_2/Z_2) \quad (6)$$

where  $l_2$  is the thickness of the denser medium. According to this expression, radiation in a single plate is four times as large as that of single interference when interference is constructive.

For a single electron traversing  $2M$  layers ( $M$  layer of medium 1 and  $M$  layers of medium 2), the factor for coherent summation from each layer with photon absorption is <sup>11,12</sup>

$$F_3 = \frac{1 - e^{-\mu L}}{\mu} \quad (7)$$

where the average X-ray absorption coefficient  $\mu = (\mu_1 l_1 + \mu_2 l_2)/l$ , the spatial period  $l = l_1 + l_2$ ,  $l_i$  ( $i=1, 2$ ) are the thickness of layers for the two media and total length  $L = Ml$ . For small absorption,  $F_3$  is proportional to total length  $L$  or  $M$ .

With assumptions that the interaction between electrons inside the beam is negligible, the average energy loss of electrons is owing to the decrease in number of electrons with distance travelled and  $M$  is sufficiently large, we obtain the formula for the number of photon generated per unit photon energy in eV per electron for a small angle  $\Delta\vartheta$  about the resonant angle  $\vartheta_n$ . Eqn (1), as

$$\frac{dN}{d\omega} = \begin{cases} (dN/d\omega)_{cr} \left( (1 - e^{-\mu L})/\Gamma - (\mu L + e^{-\mu L} - 1)/\Gamma^2 \right) & \text{for } L \leq L_{cr} \\ (dN/d\omega)_{cr} (e^\Gamma - 1 - \Gamma) e^{-\mu L} / \Gamma^2 & \text{for } L > L_{cr} \end{cases} \quad (8)$$

where  $(dN/d\omega)_{cr} = \frac{16\alpha(\Delta\varepsilon\beta)^2}{\pi\omega} |G(\beta, \varepsilon_1, \varepsilon_2, \vartheta_n)|^2 \frac{L_{cr}}{l} \sin^2(l_2/Z_2)$ , critical

length for electron energy loss  $L_{cr} = E_0 (dE/dx)^{-1}$ , angular width of the radiation  $\Delta\vartheta = 1/M$ , average electron energy loss per unit path length  $\overline{dE/dx} = ((dE/dx)_1 l_1 + (dE/dx)_2 l_2) / l$  and  $\Gamma = \mu L$ . There are numerous ways to calculate  $dE/dx$ <sup>14</sup>. For simplicity, we use the collision loss formula<sup>9</sup>

$$(dE/dx)_i = \frac{2\pi N_A r_0^2 m_0 c^2 Z_{a_i}}{\beta^2 A_i} \rho_i [\ln((\gamma-1)^2(\gamma+1)m_0^2 c^4 / (2I_i^2)) + ((\gamma-1)^2/8 + (2(\gamma-1)+1)) \ln 2 / \gamma^2 + 1 - \beta^2] \quad (9)$$

where  $m_0$  is electron mass, ionization potential of the atom  $I_i (i=1,2) = 9.73Z_{a_i} + 58.8Z_{a_i}^{-0.79}$  eV (for atomic number  $Z_{a_i} > 13$ ),  $\rho_i (i=1,2)$  are densities of the media,  $A_i (i=1,2)$  are atomic mass and Avogadro number  $N_A = 6.024 \times 10^{23}$ . With Eqn (9), we find the optimal total length  $L_{opt}$  of the superlattice required to obtain a maximum possible power of radiation

$$L_{opt} = \ln((1 + \Gamma) / \mu) \quad (10)$$

For given media and electron energy  $E_0$ , the maximum radiation can be found by maximizing  $G(\beta, \epsilon_1, \epsilon_2, \vartheta_n)$  with respect to emission angle  $\vartheta_n$  for a chosen  $n$ . And the optimal spatial period  $l$  and ratio  $r = l_2/l$  are found by the resonance condition Eqn (1) and the condition  $l_2/Z_2 = 2\pi m + \pi/2$  where  $m$  is an integer. Together with  $L_{opt}$  from Eqn (11), we find the optimal radiation with  $n$  and  $m$  set to one. Some of the results are tabulated in table I.

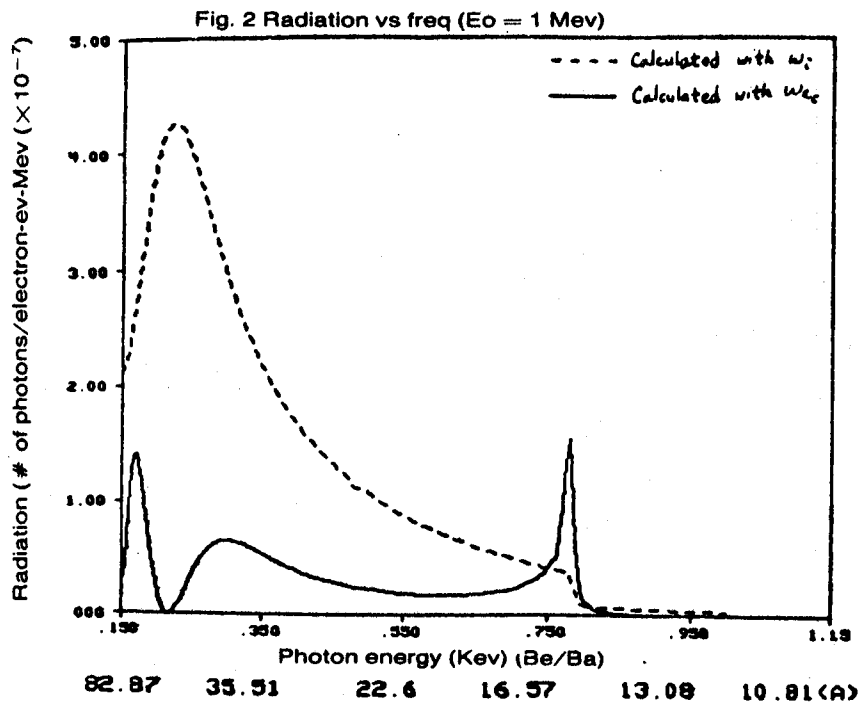
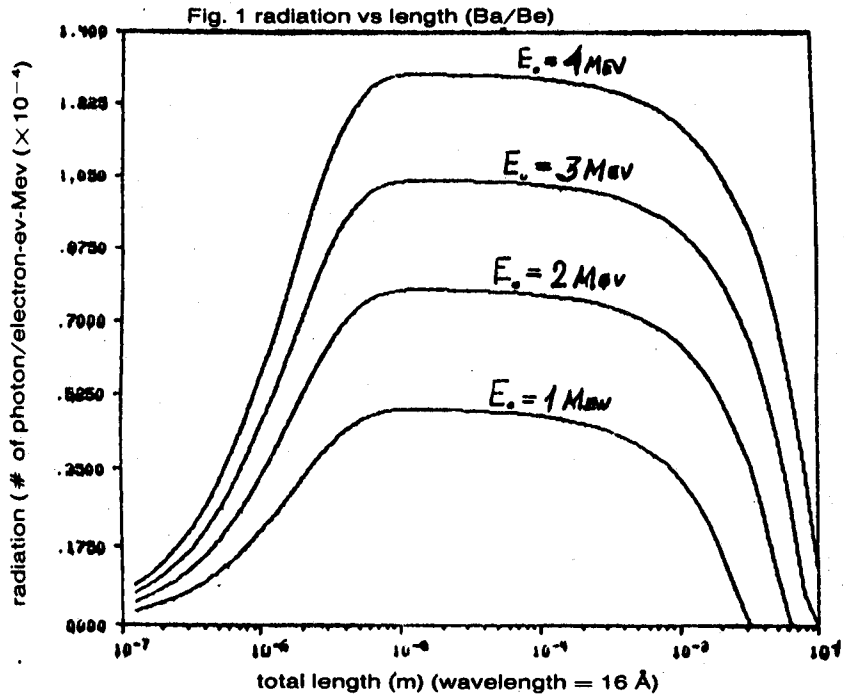
| a) $E = 1 \text{ Mev}$ Photon energy = 0.157 KeV (10.4°) |              |           |               |            |                       |
|----------------------------------------------------------|--------------|-----------|---------------|------------|-----------------------|
| Material                                                 | $l(A^\circ)$ | ratio $r$ | $\vartheta_n$ | $L(\mu m)$ | $\eta$                |
| Be/Ba                                                    | 563          | .48       | 21.1°         | .53        | $1.32 \times 10^{-7}$ |
| Be/Ce                                                    | 529          | .48       | 21.9°         | .62        | $1.17 \times 10^{-6}$ |
| Be/Ta                                                    | 555          | .48       | 21.3°         | .41        | $2.08 \times 10^{-7}$ |
| Be/Ce                                                    | 490          | .49       | 20.8°         | .52        | $1.67 \times 10^{-6}$ |
| Be/Eu                                                    | 570          | .48       | 21.0°         | .54        | $6.75 \times 10^{-6}$ |

| b) $E = 1 \text{ Mev}$ Photon energy = 0.6985 KeV (18.4°) |              |           |               |            |                       |
|-----------------------------------------------------------|--------------|-----------|---------------|------------|-----------------------|
| Material                                                  | $l(A^\circ)$ | ratio $r$ | $\vartheta_n$ | $L(\mu m)$ | $\eta$                |
| Be/Ba                                                     | 151          | .500      | 19.3°         | 11.37      | $2.16 \times 10^{-6}$ |
| Be/Ce                                                     | 150          | .499      | 19.3°         | 6.07       | $4.88 \times 10^{-9}$ |
| Be/Ta                                                     | 148          | .494      | 19.5°         | 1.44       | $1.13 \times 10^{-7}$ |
| Be/Ce                                                     | 150          | .498      | 19.3°         | 6.21       | $4.58 \times 10^{-6}$ |
| Be/Eu                                                     | 150          | .498      | 19.3°         | 6.19       | $1.11 \times 10^{-6}$ |

| c) $E = 0.5 \text{ Mev}$ Photon energy = 0.157 KeV (10.4°) |              |           |               |            |                       |
|------------------------------------------------------------|--------------|-----------|---------------|------------|-----------------------|
| Material                                                   | $l(A^\circ)$ | ratio $r$ | $\vartheta_n$ | $L(\mu m)$ | $\eta$                |
| Be/Ba                                                      | 281          | .498      | 28.0°         | .50        | $4.11 \times 10^{-7}$ |
| Be/Ce                                                      | 274          | .498      | 28.4°         | .57        | $3.77 \times 10^{-7}$ |
| Be/Ta                                                      | 280          | .488      | 28.1°         | .37        | $6.47 \times 10^{-6}$ |
| Be/Ce                                                      | 284          | .493      | 27.8°         | .48        | $5.11 \times 10^{-9}$ |
| Be/Eu                                                      | 283          | .491      | 27.9°         | .50        | $2.06 \times 10^{-6}$ |

| d) $E_e = 0.5 \text{ Mev}$ Photon energy = 0.8895 Kev (18.4°) |              |           |            |                  |                       |
|---------------------------------------------------------------|--------------|-----------|------------|------------------|-----------------------|
| Material                                                      | $l(A^\circ)$ | ratio $r$ | $\theta_n$ | $L(\mu\text{m})$ | $\eta$                |
| Be/Ba                                                         | 87.3         | .500      | 27.2°      | 10.20            | $5.86 \times 10^{-9}$ |
| Be/Ce                                                         | 67.2         | .499      | 27.2°      | 5.50             | $1.33 \times 10^{-9}$ |
| Be/Ta                                                         | 86.8         | .497      | 27.3°      | 1.31             | $3.11 \times 10^{-8}$ |
| Be/Ge                                                         | 67.2         | .499      | 27.3°      | 5.80             | $1.25 \times 10^{-8}$ |
| Be/Eu                                                         | 67.2         | .499      | 27.3°      | 5.70             | $3.02 \times 10^{-9}$ |

Table 1 Values of  $l$ ,  $r$ ,  $\theta_n$ ,  $L$  and  $\eta$  for various materials,  $E_e$  and photon energy.



82.87 35.51 22.6 16.57 13.08 10.81(A)

For comparison purpose, we introduce radiation efficiency  $\eta$   $(dN/d\omega)E_0^{-1}$  which has a unit of number of photon per photon energy in eV per electron per electron energy in MeV. Table 1 demonstrates that optimal length is a function of frequency, e.g.  $L \approx 0.5 \mu\text{m}$  at a wavelength of  $80 \text{ \AA}$  and  $L \approx 5 \mu\text{m}$  at a wavelength of  $18 \text{ \AA}$ . Since  $L_{opt}$  depends on photon absorption  $\bar{\mu}$  and collision loss  $dE/dx$ ,  $L_{opt}$  should depend on material. On the other hand, the angle of emission  $\vartheta_n$  depends on electron energy, e.g.  $\vartheta_n \approx 20^\circ$  at  $E_0$  of 1 MeV and  $\vartheta_n \approx 27^\circ$  at  $E_0$  of 0.5 MeV. Spatial period  $l$ , however, depends on both frequency and electron energy. Longer spatial period  $l$  requires high energy of electron as expected. And a longer spatial period  $l$  is required for low frequency in order to satisfy the resonance condition Eqn (1). However, there is no simple relation of radiation efficiency with respect to energy, frequency and photon absorption. This suggests that radiation efficiency depends on permittivity and photon absorption which are not shown in the table. We expect maximum wavelength of radiation that can be generated is about  $100 \text{ \AA}$  since absorption of X-ray photon at longer wavelength is larger<sup>8</sup>. The upper end of soft X-ray radiation is about  $10 \text{ \AA}$  for electron energy ranging from a few KeV to a few MeV. Although the method discuss here offers a very attractive option to obtain narrowband X-ray radiation with very low energy of electrons (see table 1), it is still important to compare its radiation efficiency to other methods<sup>13</sup> which use high electron energy (e.g. 100 MeV). This comparison (see table 1) reveals a striking fact that the difference in the radiation efficiency ( $\eta \approx 10^{-7}$  in Ref [11]) between these two method in typical case is not greater than one order of magnitude. Since the superlattice X-ray radiation is exploiting such a low energy of electron (200 KeV - 1 MeV) which is within the reach of ordinary university laboratory equipment, its advantage becomes even clearer.

We demonstrate the effects of photon absorption and permittivity on Fig. 1 and Fig. 2. respectively. Fig. 1 depicts the variation of radiation efficiency with total length  $L$ . The absorption of photon makes the radiation efficiency to be saturated in a short length  $L \approx 1/\bar{\mu}$ . Then the scattering of electron take over when  $L \approx L_{cr}$ . The radiation efficiency drops abruptly when  $L > L_{cr}$ . Fig. 1 demonstrates that photon absorption is dominant over electron scattering. The effect of photon absorption is even stronger in heavy material with high atomic number<sup>8</sup>. Fig. 2 depicts the results of calculating radiation efficiency versus photon energy with plasma frequency  $\omega_i$  and the parameter  $\omega_a$  [see Eqn (4) and succeeding text]. With  $\omega_a$ , the spectral shape changes drastically. This shows that the usual practice of using plasma frequency to calculate permittivity<sup>3,10,12,13</sup> is not always applicable to the soft X-ray frequency range. For light element (Lithium, Beryllium and Boron), we can still apply  $\omega_i$ . As shown in Fig. 2 for the combination of Barium and Beryllium, another narrow spectral peak is created when  $\omega_a$  is utilized. Such a narrow spectral width is caused by the anomalous dispersion near the absorption edges and can occur in other combination like Cerium with Beryllium. The generation of very narrow spectral peaks of X-ray radiation which are due to anomalous dispersion near the edges will be discussed somewhere.

### Conclusions

We have considered the role of photon absorption and electron scattering in X-ray emission by electrons with low energy (few Kev to few Mev) which are sufficient to produce resonant transition radiation in a superlattice with short spatial period ( $50 \text{ \AA}$  to  $500 \text{ \AA}$ ) proposed earlier<sup>4,5</sup>. We have formulated a numerical procedure to calculate maximum radiation with the electron energy, combination of material, and desired frequency specified. During the calculation process, the spatial period, optimal total length, ratio  $r$  of the thicknesses of each medium, and resonant angle are evaluated. This completely specifies the design of the system. For example, we obtain  $\theta \approx 150 \text{ A}^\circ$ ,  $\vartheta_n \approx 20^\circ$ ,  $r \approx 0.5$  and  $L_{opt} \approx 5 \mu m$  for a chosen  $E_0 = 1 \text{ Mev}$  and desired wavelength about  $18 \text{ A}^\circ$ . We further compare our system to other systems<sup>13</sup> employing higher electron energy (100 Mev). It turns out that the difference of the two system is about one order of magnitude for most cases. This makes our system (low electron energy plus superlattice) preferable as an inexpensive narrowband X-ray source.

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