Intensity and polarization nano-patterns in a 2D system of interacting dipoles

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Abstract: We demonstrated that predicted by us phenomenon of sub-wavelength nano-stratification of local field is feasible not only for 1D, but also for 2D-structures, where it can produce huge field enhancement via size-related resonances. ©2008 Optical Society of America

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In our other paper [1] we outlined the general theory of the new phenomenon of nano-stratification of local filed in low-dimensional structures and reported detailed results on the new effect in 1D systems. In this paper we report our pilot results of applying the general theory to 2D systems.

It is well known that in a dense medium the local field $\vec{E}^{\text{loc}}$ acting on individual atoms can differ significantly from the macroscopic field $\vec{E}$. In the simplest form, the relation between these two fields is given by the Lorentz–Lorenz formula. The knowledge of the local field is required to properly describe most optical processes, but is especially important in nonlinear optics, where it can significantly increase the efficiency of many processes or change material parameters. The Lorentz–Lorenz formula was extended and modified numerous times to account for different aspects of the inter-atomic interactions. Still, to the best of our knowledge, all these generalizations assumed a quasi-uniform response of the medium to the applied ("external") electric field $\vec{E}^{\text{ext}}$. In our present work, following [1], we lift this limitation and allow for sub-wavelength (*i. e.*, nano-scale) variations in $\vec{E}^{\text{loc}}$.

We show that under certain conditions, distinctive sub-λ spatial patterns in $\vec{E}^{\text{loc}}$ (*nano-strata*) emerge in a system of interacting atoms arranged in a planar 2D lattice. Atoms at near-resonance conditions present special interest, as the strength and sign of their dipole–dipole interaction can be easily controlled in the vicinity of an atomic resonance by tuning λ. This gives us a convenient way of controlling the emerging nano-strata. Surprisingly, these nano-scale patterns arise even if we only consider *linear* (with respect to the electric field) dipole–dipole interaction between the atoms. The nano-strata in the 2D system are reminiscent of (and related to) the spatial stratification in a 1D chain of dipoles [1].

We performed numerical simulations of the local field in a planar arrangement of atoms forming a finite close-packed triangular lattice, every atom being regarded a dipole. To simplify our computations, only the interactions between the nearest atomic dipoles in the lattice were taken into account. The results of our analytical and numerical studies of the related 1D problem suggest [1] that the nearest-neighbor approximation provides a reasonably good description of a system of interacting dipoles, giving results that are qualitatively similar to the full solution involving interactions between more distant dipoles. We observed the formation of nano-strata in the local field $\vec{E}^{\text{loc}}$ with the scale much shorter than any inhomogeneity in the applied field $\vec{E}^{\text{ext}}$. The formation of such sub-λ nano-strata is usually associated with some broken symmetry in the lattice, such as, *e. g.*, a boundary or a defect; however, the sub-λ nano-strata can extend far beyond such location. The spatial scale of the nano-strata is determined by the strength of the dipole–dipole interaction characterized by the dimensionless parameter Q, which can be adjusted by detuning the laser frequency ω from the resonant frequency ω_0 of the atomic resonances. In a finite 2D lattice of atoms, a strong field enhancement can occur in the system at some "size-resonant" values of Q, with the period of the nano-strata imposed by the number of atoms between the lattice boundaries.

It is interesting to compare the nano-strata formed in the 2D lattice of dipoles with the nano-strata emerging in a 1D chain of dipoles. For better comparison with the 1D problem, we used a 25×29 triangular lattice of atoms, with periodic boundary conditions applied at the more "ragged" edges of the lattice; the uniform external electric field $\vec{E}^{\text{ext}}$ was parallel to the "open" boundary. We varied $Q \equiv (4|d|^2)/(\varepsilon\hbar\ell^3\Delta\omega)$ [d is the atomic dipole moment, $\Delta\omega$ is the resonance detuning, ε is the (non-resonant) background dielectric constant, and ℓ is the spatial period of the lattice] in a certain range of negative values and observed strong size-related resonant enhancement of $\vec{E}^{\text{loc}}$ at certain values of Q. The solid line in Fig. 1 shows the dependence of the maximum amplitude of $\vec{E}^{\text{loc}}$ *vs* Q for the damping factor of 0.001. The periodic boundary conditions only allow formation of sub-λ nano-strata with the wave-vector $\vec{q}$ perpendicular to $\vec{E}^{\text{ext}}$. The spatial period of the nano-strata $2\pi/q$ (which we express in the units of ℓ) is determined by Q; the corresponding dependence is shown in Fig. 1 as the dashed line. We can notice that the range of Q where the resonant enhancement of $\vec{E}^{\text{loc}}$ is possible is prescribed by this dispersion curve. To find this dispersion dependence,

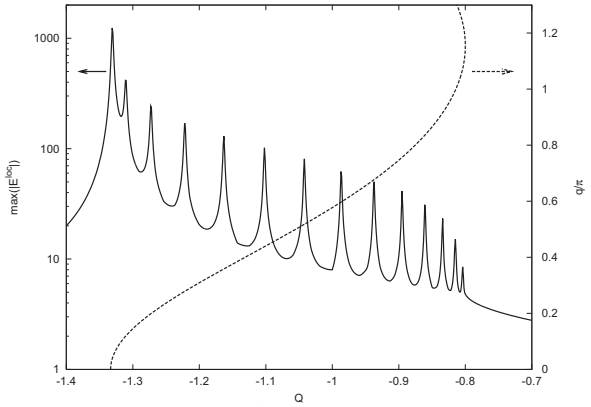

Fig. 1. Resonant behavior of $|\vec{E}^{\text{loc}}|$ as a function of Q (solid line) and the "dispersion dependence" of the emerging nano-strata (dashed line) in a 2D triangular lattice of atoms with a periodic boundary condition.

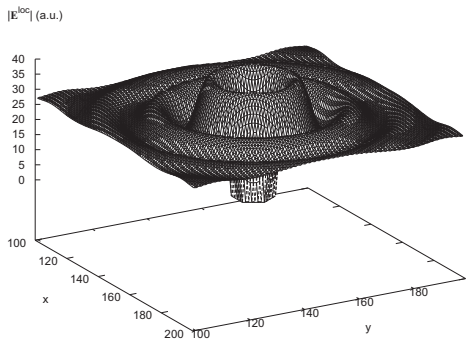

Fig. 2. The distribution of the local field $\vec{E}^{\text{loc}}$ in the vicinity of a circular hole in a planar triangular lattice of dipoles. Both $\vec{E}^{\text{ext}}$ and $\vec{E}^{\text{loc}}$ are perpendicular to the plane of the lattice. The distances along the x- and y-axes are normalized to the spatial period of the lattice.

we developed an analytical model in the approximation of the nearest-neighbor interactions between the dipoles in the lattice. In the case of the close-packed triangular lattice, every dipole interacts with its six neighbors. Based on this configuration, we found the dispersion dependence of Q *vs* the wave-vector of the nano-strata within the 1-st Brillouin zone; the dispersion curve in Fig. 1 shows this result for the direction corresponding to our chosen geometry. The results of our numerical simulations are in good agreement with the predictions of this analytical theory.

The size-related resonances shown in Fig. 1 occur at those Q, where an integer number of the half-strata-wavelengths π/q fits the width of the finite lattice. We can thus say that the nano-strata formed in the 2D lattice of dipoles are similar to the nano-strata emerging in a 1D chain of dipoles [1], but an important difference is that such resonances now occur only in a well-defined range of $|Q|$ below the main resonance, while in the 1D case it happens at $|Q|$ above that resonance.

We found that the emerging nano-strata strongly depend on the polarization of the applied field $\vec{E}^{\text{ext}}$. The case of $\vec{E}^{\text{ext}}$ perpendicular to the lattice is the simplest one, as the local electric field $\vec{E}^{\text{loc}}$ is also uniformly polarized in the same direction. Consider a standing wave formed by two electromagnetic waves counter-propagating in the plane of a finite triangular lattice of near-resonant atoms. The size of our system is about 300×300 lattice periods; a maximum of the electric field $\vec{E}^{\text{ext}}$ of the standing wave is centered on it, $\vec{E}^{\text{ext}}$ being perpendicular to the lattice; $\vec{E}^{\text{ext}}$ can be considered uniform throughout the lattice. We remove some atoms from the center of the lattice, making a roughly circular hole of 15 lattice periods in diameter. The spatial distribution of the local field $\vec{E}^{\text{loc}}$ in the vicinity of the hole is shown in Fig. 2. For this Figure, we have taken $|Q| = 0.69$ and the damping factor of 0.01. The hole in the Figure is clearly recognizable by the dip in the magnitude of $\vec{E}^{\text{loc}}$. The magnitude of the applied electric field $|\vec{E}^{\text{ext}}| = 1$ in our arbitrary units; the large relative values of $\vec{E}^{\text{loc}}$ at a distance from the hole show strong local-field enhancement for the selected Q. A rotationally-symmetric sub-λ nano-pattern in $\vec{E}^{\text{loc}}$ provides for additional local-field enhancement in the vicinity of the hole; the nano-pattern fades out at a distance from the hole.

Our results demonstrate that the new phenomenon of nano-stratification shown by us [1] to be strongly pronounced in 1D systems, remains valid for 2D systems too. In particular, we can see giant field enhancement in nano-strata for certain configurations of the lattice and field polarization. This broadens up opportunities for experimental observation of the effect by using many available 2D-like systems and indicate potential for applications in integrated optical nano-devices.

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[1] A. E. Kaplan and S. N. Volkov, "Nano-stratification of local field and atomic bistability in low-dimensional structures", submitted to this conference.