NONLINEAR WAVES AND SWITCHING EFFECTS AT NONLINEAR INTERFACES

A. E. Kaplan
Department of Electrical and Computer Engineering
The Johns Hopkins University
Baltimore, MD 21218, USA

P. W. Smith and W. J. Tomlinson
Bell Communications Research
Red Bank, NJ 07701-7020, USA

1. INTRODUCTION

Over the past several years, a new direction has arisen in nonlinear optics, which can be described as self-action at nonlinear interfaces (i.e., an interface between linear and nonlinear transparent media). In the most studied configuration, light is incident from a linear medium (with susceptibility \(\chi\)) onto the plane surface of a nonlinear medium with a nonlinear refractive index (Fig. 1). The studies of such an interface have followed an almost classical progression of alternating theoretical and experimental advances, which have resulted in an increasingly detailed understanding of the behavior, features, and limitations of nonlinear interfaces, and which have brought into focus the remaining unsolved problems.

Studies of the behavior of nonlinear interface originated in theoretical analyses\(^\text{1-4}\) which assumed that the incident and reflected fields (i.e., the fields in the linear medium) can be represented as infinite plane waves and that the field in the nonlinear medium is either an infinite plane wave or a nonlinear evanescent wave, or a mixture of both. These studies resulted in closed-form analytical expressions. We refer to them as the "plane wave theory". A major result of the plane-wave theory was that under appropriate conditions the reflectivity of a nonlinear interface would exhibit a threshold behavior as a function of the intensity of the incident field; for intensities below the threshold value the interface would be totally reflecting, and for intensities above the threshold the interface would have a significant transmission. The plane-wave theory also predicted that for a range of input intensities just below the threshold value the reflectivity of the interface can be bistable.

The behavior predicted by the plane-wave theory suggested several applications (e.g., ultra-fast optical switches, scanners, and logic elements), and experiments were initiated to study these predicted phenomena. The initial experimental results were obtained using Cs\(_2\) as the nonlinear medium (with the so-called positive or self-focusing nonlinearity, see below), and a high-power mode-locked ruby laser as the source.\(^\text{5-9}\) The experimental results clearly displayed the predicted threshold behavior. They also seemed to show certain evidence of the predicted bistability.\(^\text{1,2}\) However, because of the limited time resolution of the detection system, and the fact that the experiments were done with only a single input pulse width, this evidence was inconclusive.

The interpretation of the experimental results was also complicated by the fact that the experiments were done with a moderately tightly focused Gaussian beam (not an infinite plane wave). Various numerical simulations were reported for a one-dimensional Gaussian input beam (i.e., a beam with a Gaussian distribution in the plane perpendicular to the interface, but which extends uniformly to infinity in the direction perpendicular to the plane of
incidence)\(^{9,10}\) with contradictory results. Some of them (Ref. 10 and to some extent Ref. 11) endorsed the existence of bistability for the Gaussian beam case, while others\(^{11}\) disputed it. Although some numerical simulations\(^{12,13}\) predicted the appearance of nonlinear surface waves, later results\(^{14,15}\) showed that these surface waves are prohibited under conditions implied in Refs. 8 and 10-12. The most recent modeling study confirmed the threshold behavior, but showed that the detailed behavior of the interface is quite complex.\(^{16}\) The reflectivity exhibits not a single threshold, but a series of thresholds at increasingly higher input intensities. The transmitted light was shown to form self-trapped channels, which can easily be identified with the two-dimensional self-trapping\(^{17,18}\) well known in the theory of light propagation in a semi-infinite medium with a positive nonlinear refractive index. In the case of grazing incidence, though, these channels propagate along complicated curved paths, with each step in the reflectivity associated with the formation of an additional channel. The simulations used a steady-state model, which is not capable of proving, or disproving, the existence of bistability, but it was postulated that the nonlinear interface could not exhibit bistability, because of the lack of any memory mechanism to store the information on the previous field configuration.\(^{19}\)

The next set of experiments made use of an "artificial" nonlinear medium, with a very large, but also very slowly responding, effective nonlinearity.\(^{20}\) This made it possible to do experiments with a cw laser, and relatively detailed data were obtained on the reflectivity of a nonlinear interface as a function of the input beam intensity. The results clearly displayed the multiple thresholds, and were in good agreement with the (one dimensional) Gaussian beam simulations. Bistability was not observed. However, it was found that when the input intensity was lowered through a threshold value, the high transmission state would persist for of the order of 300 times the response time of the nonlinearity. This bistability has not yet been accounted for theoretically. Hysteresis and, in fact, quasi-stability have also been reported recently\(^{21}\) in an experiment on the nonlinear reflection of picosecond pulses from a glass-liquid interface; the authors of Ref. 21 attribute this effect to probable thermal changes in the near-surface layers in both the glass and the liquid.

The case of a negative (self-defocusing) nonlinearity has been much less thoroughly explored in experiments, but for this case, the plane-incident-wave theory predicts\(^{22,23}\) an excitation of nonlinear waves of a new kind (described in Refs. 3-5 as longitudinally inhomogeneous traveling waves). This poses interesting physical features.

All of the nonlinear interface effects are caused by the competition between the mismatch in linear susceptibilities \(\Delta n\) and the nonlinear component \(\Delta n_{\text{NL}}\). If the linear difference is sufficiently small, this competition can result in a strong change of reflection even if the nonlinear component is small as well (as it usually is in an optical range). As the intensity of the incident field increases, the penetrating (or evanescent) field in the nonlinear medium changes the index difference at the interface. At a critical intensity, an abrupt switch from total internal reflection to partial transmission occurs if the nonlinearity is positive, or vice versa if the nonlinearity is negative. In order for these effects to occur, it is necessary to have \(\Delta n_{\text{NL}} < 1\), \(\phi < 1\) (see below, Section 3).\(^{24}\) The lack of these conditions leads to absence of the phenomenon. This may be the reason why such effects were not discovered earlier, either in theoretical or experimental research, e.g. in the work\(^{25}\) devoted to harmonic generation due to reflection of light from nonlinear dielectrics.

The optical hysteresis and bistable behavior suggested by the plane-wave theory\(^{26}\) can be viewed as phenomena quite typical for many nonlinear systems. The concept of bistability (or multistability, in general) implies that the system can assume one of two (or more) stable stationary states under the same external conditions. Usually it correlates with hysteresis which occurs as one of the external parameters of the system (e.g. the intensity of incident radiation) vary. This phenomenon is well known in physics, e.g. in the field of phase transitions, in the theory of nonlinear mechanical or electronic oscillators,\(^{27}\) etc.

One of the reasons that the predictions of bistable and hysteretic reflectivity at a nonlinear interface attracted attention is that in recent years optical bistability has become a very promising field in nonlinear optics.\(^{28}\) Elements based on optical bistability can be utilized for optically-controlled memory and logic operations in high-speed optical signal processing systems.\(^{29}\) Many known bistable optical devices comprise a Fabry-Perot resonator filled with a nonlinear medium, as first proposed in Refs. 26 and 27; bistability in these systems was first observed in Ref. 28. In these systems, bistability is due to the presence of a resonator which provides a feedback. The media used might have a resonant saturated absorption\(^{30,31}\) or a non-resonant Kerr nonlinearity.\(^{32}\) The use of Fabry-Perot resonators causes these devices to
be strongly selective to the frequency of the incident light. In contrast to these devices, a nonlinear interface does not employ any resonators. Therefore, no resonant tuning of the laser frequency is required; moreover, a broad spectrum of input light can be used.

A number of new effects, which do not exist in resonators, have also been predicted for nonlinear interfaces, and have stimulated various proposals for applications. New optical switching devices related to the nonlinear interfaces have been proposed in Refs. 6 and 9. It was suggested in Refs. 1 and 2, and studied in detail in Ref. 30, that nonlinear interfaces should be used as bistable mirrors for self-pulsing lasers. Most recently, reversible optical computing circuits based on the plane wave switching effect at nonlinear interfaces were proposed.32

Interesting modifications of the nonlinear interface idea have appeared, including optical bistability in a frustrated-total-reflection optical cavity,33 and in a dielectric cladded thin film near the total internal reflection state.34 An enhancement of nonlinear interface effects by exploiting surface plasmons at metallic surfaces was proposed in Ref. 34 and experimentally explored in Ref. 35. Optical bistability using long-range surface plasmons was proposed in Ref. 36, and surface resonance modes (attributed to dielectric spheres placed on a metal substrate) in Ref. 37. An interesting analogy of bistable total internal reflection at nonlinear interfaces was found38 in the bistability of the reflection of light from metallic diffraction gratings (positioned at a nonlinear interface) in the vicinity of the Wood anomaly. One trend in the theory and applications of nonlinear interfaces is the study of nonlinear waveguides, e.g., dielectric waveguides formed by a linear dielectric layer sandwiched between two nonlinear layers or vice versa (see e.g., Refs. 9, 39-41 and references therein), which have promising characteristics.

Despite its deficiencies, the plane-wave theory of nonlinear interfaces has proven to be a useful tool, and a powerful stimulant to research in this field. The theory has suggested a number of interesting nonlinear effects:

(i) multistability and hysteresis jumps in the reflection coefficient;3,4,43,45
(ii) change and scattering of the reflection angle and reflection coefficient by varying the intensity of the incident light;3,4,45
(iii) optically-induced transparency of the interface by incident light with specific intensity;3,4,45
(iv) change of penetration depth of the field into a reflecting medium in the regime of total internal reflection;3,4
(v) excitation of nonlinear waves of a new kind3,4 (longitudinally inhomogeneous traveling waves (LITW)) and effects related to longitudinal inhomogeneous traveling waves, namely:
(vi) strong nonlinear self-parallels of refracted rays along the interface; and (vii) self-limiter of the energy flux of the refracted light.4,43

The theory has also been used to describe other situations, for example bistability at an electro-optically driven interface,43 which has recently been observed experimentally,44 and bistable reflection of light at a nonlinear-optical waveguide junction.4

The main results of the plane-wave theory are described in Section 2. The effects predicted by this theory in the particular cases of positive and negative nonlinearities are described in Sections 3 and 4, respectively. The experimental and computer simulation results are discussed in Section 5.

2. PLANE-WAVE THEORETICAL FORMALISM

Let a plane wave with amplitude $E_0$ be incident from a linear medium with susceptibility $\varepsilon_0$ at the glancing angle $\theta$ (Fig. 1) onto the boundary of a nonlinear medium whose susceptibility, $\varepsilon_1$, depends on the field amplitude $E$ in the medium according to the equation:

$$\varepsilon_{NL}(E) = \varepsilon_0 + \Delta\varepsilon_1 + \Delta\varepsilon_{NL}(E)^2$$

(1)

where

$$\Delta\varepsilon_{NL}(E)^2 = \varepsilon_1 E^2$$

(2)

$\varepsilon_1$ is the nonlinear coefficient (this is usually referred to as a Kerr nonlinearity), and $\Delta\varepsilon_1$ is the field-independent mismatch between susceptibilities of the two media. The most interest-
ing self-action effects are expected when the nonlinear component $\Delta \nu_{NL}$ (even if it is small) can compete with the linear mismatch $\Delta \nu_{L}$. Only then can the nonlinear component (caused by the field penetrating into the nonlinear medium) dramatically change the total difference $\Delta \nu = \Delta \nu_{L} + \Delta \nu_{NL}$ between the susceptibilities of the two media. Such a change, in turn, can result in an abrupt jump of the reflectivity of the interface at some critical incident intensity, e.g., the switch from the total internal reflection state into the transmission state. For the same reason, the incidence of the light has to be almost grazing, because the required magnitude of the glancing angle $\psi$ should be of the order of the critical angle for total internal reflection, which results in the following conditions:\(^{1,4}\)

$$\left| \frac{\Delta \nu_{NL}}{\nu_{L}} \right|, \left| \frac{\Delta \nu_{NL}}{\nu_{NL}} \right| \ll 1 \quad \psi \ll \left( \frac{\nu_{NL}}{\nu_{L}} \right)^{1/2} \ll 1$$  \hspace{1cm} (3)

These conditions give $\psi \sim 1$ for $|\Delta \nu_{NL}/\nu_{L}| \sim 10^{-8}$. For most of the rest of this chapter we will assume that conditions (3) are valid. This will simplify the theory and make the phenomena independent of the polarization of the incident light.\(^{1,4}\)

The field in the linear medium is represented in the form of two homogeneous plane waves, one of which is the incident wave and the other is the reflected wave, determined by an unknown complex reflection coefficient $r$. The wave equation for the complex amplitude of the field, $E$, in a nonlinear medium in the two-dimensional case is written as

$$\frac{d^2 E}{dx^2} + \frac{k_0^2}{c^2} \left[ \frac{\nu_{NL}^2}{\nu_{L}^2} \right] \cos \theta \sin \theta = 0$$  \hspace{1cm} (4)

where $k_0 = \omega\sqrt{\epsilon_0}/c$, and the z-axis is taken to be perpendicular to the boundary. The total field is $E(x) = E_{inc} + r E(r)$. By equalizing tangential components of the fields on both sides of the boundary, the generalized boundary condition for the reflected wave $E$ can be obtained:\(^{1,4}\)

$$\frac{d E(r)}{dx} + k_0 \nu_{NL}(E_{inc} - E(r)) = 0$$  \hspace{1cm} (5)

The expression for the reflection coefficient $r$ is

$$r = \frac{E(0)}{E_{inc}}, \quad 1$$  \hspace{1cm} (6)

which also follows from the continuity of tangential components of the fields at the interface. The boundary conditions are a very important part of any problem related to nonlinear interfaces. The failure to define these conditions adequately could result in missing important effects. For example, inadequate conditions for the reflected plane wave in an earlier work\(^{45}\) made it impossible to determine the wave amplitude and refraction angle separately in the
case of $s > 0$. This led the authors of Ref. 46 to controversial conclusions regarding the existence of optical bistability even within the plane-wave approach.

The field $E$ in the nonlinear medium is written in the form\textsuperscript{8,9}

$$E = u(x) \exp \left[ \frac{i}{\hbar} \int_{x_0}^{x} \left( \frac{\partial u}{\partial t} + i \xi u \right) dx \right]$$

(7)

where $u(x)$ and $\xi(x)$ are real quantities, $u$ being the real amplitude of the field, and $\xi$ being the angle between the rays and the $x$-axis at a given point $x$; $\xi$ is a constant phase term. For plane waves, $u$ and $\xi$ are constant, and for surface (or evanescent) waves, when total internal reflection occurs, $\xi = 0$. In general, $u$ and $\xi$ are not constant.

In the theory of reflection from a semi-infinite medium, the conditions at infinity must be satisfied. Since there are no sources inside the nonlinear medium, traveling waves should only propagate away from the interface in the nonlinear medium (surface waves do not carry any energy along the $x$-axis at all). Therefore, in Eq. (7) for $x \rightarrow \infty$, the following conditions have to be satisfied\textsuperscript{13,14}

$$u \rightarrow \text{const} \times u_{\infty} \equiv \text{const} \times \xi_{\infty} \equiv \text{const} \geq 0$$

(8)

which is, essentially, the Sommerfeld radiation condition (the absence of the backward traveling wave). Substituting the field in the form of Eq. (7) into the wave equation [Eq. (4)], we obtain its first integral

$$I = \int_{x_0}^{x} \left( \frac{\partial u}{\partial t} + i \xi u \right) dx \geq 0$$

(9)

(which expresses the conservation of the energy (flux), as well as the equation for the real amplitude $u$,

$$\frac{d^2 u}{dx^2} + k_0^2 \frac{\sinh(u)^2}{e_0} - \cos \xi \cdot \frac{T^2}{u^2} = 0$$

(10)

The first integral of this equation, which satisfies the radiation condition [Eq. (8)], can be written in the form:\textsuperscript{5}

$$\int_{x_0}^{\infty} \left( \frac{d^2 u}{dx^2} + k_0^2 \frac{\sinh(u)^2}{e_0} - \cos \xi \cdot \frac{T^2}{u^2} \right) dx = 0$$

(11)

where the nonlinear "characteristic" function $F(u)$ is introduced:

$$F(u) = u^2 \left[ \frac{\sinh(u)^2}{e_0} - \cos \xi \right]$$

(12)

Integrating Eq. (11) and taking the boundary condition Eq. (5) into account, one can obtain all possible wave solutions for the problem.

3. POSITIVE NONLINEARITIES

In the simplest case of cubic nonlinearity [Eq. (2)] with $s > 0$ (i.e. Kerr nonlinearity), only two kinds of wave modes, with their intensities depending on $u$ only, can exist:

(i) the homogeneous plane wave, $u(x) = \text{const}$, which corresponds to the transmission regime ($|r| < 1$), and

(ii) the surface wave, or evanescent wave, $\xi = 0$, which corresponds to total internal reflection ($|r| = 1$).
Both of these waves are nonlinear analogs of corresponding linear waves, but differ from those in their dependence on the incident light intensity. We should note that in two, or more, dimensions, a homogeneous plane wave is not a stable solution in a medium with a positive Kerr nonlinearity, and thus we would only expect the plane-wave theory to apply to the transmitted field in a region sufficiently close to the interface. Using Eqs. (4)-(7) with $u$ and $\xi$ constant, we readily obtain "Snell's nonlinear formula" for the transmission angle $\psi$:

$$i + \frac{\mathcal{E}_{0}}{\mathcal{E}} \left[ i - \mathcal{E} \cdot \frac{\Delta \mathcal{E}}{\mathcal{E}} \right] - \delta \mathcal{E} \left| \mathcal{E} \right|^2 = 0 \tag{13}$$

or "Fresnel's nonlinear formula" for the reflection coefficient $r$,

$$4\mathcal{E}^2 + \frac{\Delta \mathcal{E}}{\mathcal{E}} (1 + s^2) + \frac{\mathcal{E}^2}{\mathcal{E}} |\mathcal{E}^2| (1 + s^2) = 0 \tag{14}$$

(see Fig. 2). It was shown in Ref. 5 that in the general case of arbitrary magnitude of the nonlinearity $\mathcal{E}_0$, the incident angle $\psi$ (i.e., in the case when Eq. (3) is not satisfied any more), "Snell's formula" for the angle $\psi_{\text{SN}}$ of a reflected plane wave is

$$\tan \psi_{\text{SN}} = \frac{\Delta \mathcal{E}}{\mathcal{E} \sin \psi} \tag{15}$$

(where now $\tan \psi_{\text{SN}} = (\xi/\cos \psi)$, while "Fresnel's formula" for the amplitude $u$ of a reflected plane wave is

$$4 \sin^2 \psi \left( 1 - \frac{\mathcal{E}^2}{\mathcal{E}} \right) = \frac{\Delta \mathcal{E}}{\mathcal{E} \sin \psi} \tag{16}$$

The formula for the reflection coefficient $r$ follows from Eq. (16) via Eq. (6) which again yields Eq. (14) where one has to replace $\psi$ by $\mathcal{E}$.

The nonlinear total internal reflection state can be excited if the linear mismatch is negative ($\Delta \mathcal{E} < 0$) and $\psi < \psi_{\text{SN}}$, where $\psi_{\text{SN}} = (\Delta \mathcal{E} / \mathcal{E})^{-1/2}$ is the critical angle of linear total internal reflection. The profile of wave intensity now is not exponential: it is obtained from Eq. (11) with $u_{\text{SN}} = 0$. In the case of a Kerr nonlinearity [Eq. (2)], the general equation (11) under the conditions given by Eq. (3) can be written in the form

$$\left( \frac{\partial \mathcal{E}}{\partial \xi} \right)^2 = \mathcal{E}_0^2 \left( \mathcal{E} \cdot \mathcal{E} - \mathcal{E}^2 \right)^{-1} \tag{17}$$
Integration of this equation yields the amplitude profile \( u(z) \) of a total internal reflection evanescent field (surface wave) which has a "self-channel" shape well known in the soliton theory of two-dimensional self-trapping:

\[
\frac{\text{d}u(z)}{\text{d}z} + \gamma u(z) \left[ u(z)^2 - 4|\mathbf{E}_n|^2 \right] = 0
\]

(19)

where \( \gamma = (\Delta^2 - \omega^2)^{1/2} \). The constant \( C \) is determined from the boundary conditions (Eq. 5), and can have from one to four values (for different \( N \) and \( E_n \)) which causes multistability and hysteresis jumps (see Fig. 2). Indeed, from the complex boundary condition (Eq. 5), one obtains a condition for the real amplitude \( u(0) \) of an evanescent field at the boundary:

\[
\frac{\text{d}u(0)}{\text{d}z} + \gamma u(0) \left[ u(0)^2 - 4|\mathbf{E}_n|^2 \right] = 0
\]

(20)

as well as a formula for the phases of the evanescent field \( \phi \) and reflected wave \( \phi_r \) (where \( \phi_r \) is determined by the relationship \( r = \exp(i\phi_r) \))

\[
\phi = \frac{\pi}{2} - \arctan \left( \frac{2\mathbf{E}(0)}{E_n} \right)
\]

(21)

where the upper sign corresponds to \( u(0) < 0 \) and the lower to \( u(0) > 0 \). Now, eliminating \( \text{d}u/\text{d}z \) from Eq. (19), and from Eq. (17) (taken at \( z = 0 \)), one obtains an amplitude \( u(0) \):

\[
|u(0)|^2 = \frac{\gamma}{\gamma^2} \left[ \gamma^2 + 4|\mathbf{E}_n|^2 \right]^{1/2}
\]

(22)

Specifying that the amplitude \( u(0) \), Eq. (21), and the phase \( \phi \), Eq. (20), be real quantities, one obtains the conditions for nonlinear total internal reflection to occur:

\[
0 \leq u(0) \leq 4|\mathbf{E}_n|^2
ewline
0 < \frac{\gamma}{\gamma^2} |\mathbf{E}_n|^2 \leq \frac{\gamma}{\gamma^2} |\mathbf{E}_n|^2
\]

(23)

Based on these conditions, one can show that the threshold grazing angle \( \phi_{th} \), at which the switch from nonlinear total internal reflection to partial transmission occurs, is given by

\[
\left[ \frac{\gamma}{\gamma^2} |\mathbf{E}_n|^2 \right] = \begin{cases} \frac{2\mathbf{E}(0)}{E_n} \leq |\mathbf{E}_n| & \text{for } 4|\mathbf{E}_n|^2 \leq |\Delta| \\
\frac{1}{\frac{\gamma}{\gamma^2}} |\mathbf{E}_n| & \text{for } 4|\mathbf{E}_n|^2 > |\Delta|
\end{cases}
\]

Therefore for a fixed grazing angle \( \phi \), the threshold input intensity \( |\mathbf{E}_n(t)|^2 \), at which such switching occurs, is given by

\[
\frac{\phi_{th}}{\phi} = \begin{cases} \frac{2\mathbf{E}(0)}{E_n} \leq \frac{\Delta}{\phi} & \frac{1}{\sqrt{2}} < \frac{\Delta}{\phi} < 1 \\
\frac{\Delta}{\phi} & 0 < \frac{\Delta}{\phi} \leq \frac{1}{\sqrt{2}}
\end{cases}
\]

(24)
Thus, if starting from total internal reflection at low input intensity and gradually increasing the intensity, the interface switches from the total internal reflection state ($\xi = 1$, see Fig. 2) at some intensity given by Eq. (24). The reflection coefficient $r$ at this point is determined by Eq. (14), where the value of $\xi_{\text{Int}}$ should be substituted from Eq. (24). In the reverse process (i.e., where the input intensity is decreased), switching from the partial transmission state to the total internal reflection state occurs at the lower intensity, $\xi_{\text{Crit}}$, which is determined from Eq. (13) (or Eq. (14)) by assuming $\xi = 0$ (or $r = 1$):

$$\frac{\xi_{\text{Crit}}}{\Delta \xi_{\text{crit}}} = \frac{1}{4} \left( 1 - \frac{\delta}{V_{\text{tr}}} \right)^2$$

(25)

i.e., $\xi_{\text{Crit}}^2 < \xi_{\text{Crit}}^2$, (see Fig. 2). A jump in the reflection coefficient occurs only during the switch from the total internal reflection to the transmission state; when moving in the opposite direction, there is only a jump in the phase of reflection.

One of the interesting features of nonlinear reflection is what can be called nonlinear bleaching (or optically-induced transparency).\(^{13}\) It is seen from Eq. (14) that at $\Delta \xi_{\text{crit}} < 0$, a "bleaching" input intensity, $\xi_{\text{Bleach}}$, exists:

$$\xi_{\text{Bleach}} = \frac{\Delta \xi_{\text{crit}}}{\Delta \xi}$$

(26)

at which there is no reflection at all ($r = 0$) for any glancing angle $\psi$ (see Fig. 2). This phenomenon cannot be regarded as a nonlinear analog of Brewster bleaching (since it does not depend on either the angle or polarization of incident light). The nonlinear bleaching is attributed to the fact that the field equals the susceptibility of the two media and, therefore, makes the boundary completely transparent. In this case, the jump from total reflection to total transmission occurs at $\psi = \psi_{\text{Bleach}} \approx \frac{\gamma}{2}$ (see curve 5 in Fig. 2). For $\beta_{\text{Bleach}} = 10^{-13}$ cm$^{-1}$, we have $E_{\text{Bleach}} = 1.8 \times 10^4$ V/cm at $\Delta \xi_{\text{crit}} = 10^{-3}$, in which case $\psi = 0.24$.

4. NEGATIVE NONLINEARITIES; LATERALLY INHOMOGENEOUS TRAVELING WAVES (LITW)

In a number of physical situations, the nonlinearity can be negative (i.e., $\gamma < 0$), for instance due to nonlinear resonant interactions (see e.g., Ref. 47), or due to giant nonlinearities, recently observed in some semiconductors (e.g., in InSb, Ref. 48). In such cases, the nonlinear analog of linear waves (i.e., plane waves and total internal reflection) can exist as well. For plane waves, Snell's (Eqs. (13), (15)), as well as Fresnel's (Eqs. (14), (16)), formulae remain valid (one has to take into consideration that now the sign of $\gamma$ has changed). In this regime it is also possible to vary the value of the transmission angle $\xi$ and the reflection coefficient $r$ by changing the incident light intensity (Fig. 3). Similarly to the "positive" nonlinearity, a nonlinear total "bleaching" of the interface also exists, the "bleaching" incident light intensity is given by the same expression Eq. (25) but now the value of the linear mismatch should be positive (Fig. 3, curves 3 and 4).

In contrast to the "positive" nonlinear case, the surface (evanescent) waves at "negative" nonlinear interfaces in the total internal reflection regime exist in exactly the same range of linear mismatches and glancing angles (i.e., $\Delta \xi_{\text{crit}} < 0$ and $\psi < \psi_{\text{crit}} = \frac{\Delta \xi_{\text{crit}}}{\gamma}$) as in the linear case. However, the amplitude profile of the surface waves, given by integration of Eq. (11), is now:

$$u = \left[ \frac{\gamma}{\gamma_{\text{crit}}} \right]^{1/2} \frac{1}{\sinh(\gamma' \xi/2 + C)}$$

(27)

where $\gamma = (\gamma_{\text{crit}} - \gamma')^{1/2}$, and $C$ is a constant determined by the boundary conditions.
The amplitude of the field $u(0)$ at the boundary is given by Eq. (21), where one has to take the negative sign in front of the radical. As $\xi \to \infty$, the evanescent field, Eq. (27), decreases as $\sim \exp(-K_x \xi)$, just as in the linear case. But the depth of field penetration into the medium (at the half-intensity level), $L_{4LL}$, which is determined by the amplitude profile, Eq. (27), and the value of $u(0) = u_0$, decreases.$^3$

$$L_{4LL} = L_4 \left[ 1 + \frac{2}{n^2 \omega} \left( 1 + \left[ \frac{u_0^2}{2} \right]^{1/3} \right) \right] \left( 1 + \left[ \frac{u_0^2}{2} \right]^{1/3} \right)$$

where $L_4 = (2\pi)/(2k_0 \gamma)$ is the depth of the linear penetration. In a strong field, i.e. for $u_0^2 \gg \gamma^2$, one has $L_{4LL} \approx 0.66 \xi/\eta \ll L_4$.

The most interesting feature in the case of negative nonlinearity is the feasibility of excitation (under some special conditions) of waves of a new kind, so called longitudinally inhomogeneous traveling waves, obtained analytically in Refs. 3-5. Being inhomogeneous and non-planar near the boundary, these waves reduce to plane waves sufficiently far from the boundary. The concept of longitudinal inhomogeneity was introduced in Ref. 3 to distinguish this phenomenon from the self-action effects due to transversely inhomogeneous waves, such as self-focusing$^{4-6}$ and self-bending$^{4-6}$ of beams with limited cross-section.

Examination of Eq. (11) shows that under the conditions

$$\frac{2q}{a} \left| \frac{d^2 \psi}{d\psi^2} \right| < q \quad q = \frac{\Delta \omega}{\omega} \text{ or } \text{sin}h \beta$$

excitation of longitudinally inhomogeneous traveling waves is possible. Integration of Eq. (11) yields two possible intensity profiles for longitudinally inhomogeneous traveling waves$^{4-6}$.

$$u^1 = u_0^1 \times 2B \left[ \text{sinh}^{-1} \left( \frac{q}{\Delta \omega} \right) \right] \left( Bk_0 \gamma \right)$$

$$B = \left[ \frac{\Delta \omega}{\omega} \left( u_0^1 \right) \right]^{1/3}$$
For the minimum possible energy density at infinity,

\[ u_{m_0} = \frac{u_{m_0}^*}{\frac{2}{3} p_{01}^2} \tag{31} \]

only one (limiting) type of longitudinally inhomogeneous traveling wave remains:

\[ u_{m_0} = \frac{\alpha}{\frac{2}{3} p_{01}^2} \left[ \left( \frac{1}{k_0^2} + \frac{1}{C_2^2} \right) \right] \tag{32} \]

A unique "continuum" problem arises in relation to the new solutions: we have only two conditions [which are equivalent to a single complex boundary condition, Eq. (31)] to determine three unknown constants \( b_m \), and \( C \) in Eq. (32), and \( \varphi \) in Eq. (33). The radiation conditions, Eq. (9), have already been used for the construction of the longitudinally inhomogeneous traveling waves, Eqs. (11) and (30). Therefore, there is a continuum of solutions even when the boundary and radiation conditions are completely specified.

This situation differs drastically from the situation in a linear medium and a "positive" nonlinear medium. An energy criterion for selecting a unique kind of physically realized longitudinally inhomogeneous traveling wave was proposed by an asymptotic method whereby a weak absorption is introduced and the asymptotic results are obtained when the absorption tends to zero. A small parameter of this limiting transition turned out to be \( k_0 [k_0^2 + C_2^2] \), where \( L_0 \) is a characteristic length of the wave absorption.

Now, when the unique "surviving" type of longitudinally inhomogeneous traveling wave (but not yet unique solution) is found, one can obtain all the characteristics of these waves and, in particular, their amplitude at the boundary, \( u_a \). Based on this result and using the relationship for \( u_a \) for plane waves, Eq. (16), it is easy to find the domains of existence of all of the possible waves (i.e., plane wave, longitudinally inhomogeneous traveling wave and total internal reflection) in the system. These domains are shown in Fig. 4 in the space of the generalized parameters \( P \) and \( Q \):

\[ P = \frac{\alpha [E_0^*]_o}{c_0 \sin \psi} \quad Q = \frac{\alpha \Delta_k}{c_0 \sin \psi} + 1 \tag{33} \]

The longitudinally inhomogeneous traveling wave excitation gives rise to new effects. First of all, as in the "positive" nonlinear case, it leads to the hysteretic behavior of a system under definite conditions, but now the hysteresis jumps occur between states of two different traveling waves, plane waves and longitudinally inhomogeneous traveling waves (in contrast to "positive" nonlinearity, where they occur between plane waves and total internal reflection). Hysteresis occurs only under the condition:

\[ |p_2| [E_0^*]_o^2 > \Delta_k > 2 \alpha_0 \sin \psi \]

The behavior of the reflectivity for a system with several different parameters is shown in Fig. 3 (in particular, one can see hysteresis, curve 5, and self-focusing, curves 3 and 4). In this figure, \( E_{c}^* \) is the critical intensity of the incident light required for excitation of longitudinally inhomogeneous traveling waves for each particular case, and \( D \propto D_0 / c_0 \).

Another effect, which is of interest for applications, consists of the self-focusing of the energy flux of longitudinally inhomogeneous traveling waves (in the photonic medium). This is a direct consequence of the selection of a unique value \( u_a = u_m \), Eq. (33). The behavior of the energy flux \( I \) is shown in Fig. 5 for hysteretic and non-hysteretic situations.

One of the most interesting effects attributed to the excitation of longitudinally inhomogeneous traveling waves is self-focusing, i.e., a displacement of the refracted rays along the interface. This effect is not a nonlinear analog of Gouy-Hänsch displacement of Gaussian...
beams under total internal reflection. This is because, in contrast to the Goss-Hütchen effect, self-parallax is now valid for plane waves and for the transmission regime, rather than for total internal reflection. It corresponds to the initial "sliding" of the rays to the interface (Fig. 1, curve 2) which occurs because of conservation of the energy flux (Eq. (9): the angle $\xi$ should increase as the intensity of decrease), and it is not related to the existence of hysteresis. The ray trajectory in space is determined by the equation:

$$x = z \left( \frac{1}{Q} \right)^{1/5} \left( 1 + \frac{2z}{7} \right)^{1/5}$$  \hspace{1cm} (34)

where $\delta$ is the parallax (ray displacement) for $z \to \infty$ and $z_0$ is the characteristic depth for half of the parallax; $\delta$ and $\alpha_0$ are determined by $E_{0\gamma}$, $\psi$, and $\omega_L$. Let $k_0 = 10^4$ cm$^{-1}$, $\Delta k/L_f = -10^{-7}$, $\psi = 0.07$ and $|\psi|/|\omega_L| = 10^{-12}$, the critical value of $E_{0\gamma}$ for excitation of longitudinally inhomogeneous traveling waves is then $E_{0\gamma} = 1.15 \times 10^9$ V/cm, and if $E_{0\gamma} = 2E_{0\gamma}$, one obtains $\delta = 1.2$ cm and $z_0 = 0.01$ mm; the parallax is very strong and occurs at a very small depth.

The existence of longitudinally inhomogeneous traveling waves in a system with "negative" nonlinearity and its absence in a "positive" nonlinear system poses a number of questions concerning the kind of nonlinearity that may permit longitudinally inhomogeneous traveling waves. For example, is it possible to predict what kinds of nonlinearities and system characteristics give rise to longitudinally inhomogeneous traveling waves and what kinds of nonlinearities do not allow them? What are the parameters of longitudinally inhomogeneous traveling waves if these are allowed?

These questions have been addressed by a theorem formulated and proved in Refs. 3 and 4. This theorem relates the existence of different kinds of traveling waves to the behavior of a "characteristic" nonlinear function $F(u)$, Eq. (12) above. The main conclusion of this theorem...
is that the existence of longitudinally inhomogeneous traveling waves in a transparent medium is possible if, and only if:

(i) There is a range \( V \), for which \( \mathcal{F}(u) > 0 \), \( u \in V \).

(ii) \( \mathcal{F}(u) \) decreases at least somewhere in \( V \) (i.e., there is at least one interval \( W \subset V \) where \( \mathcal{F}(u) \) falls monotonically).

(iii) If these conditions are satisfied, the longitudinally inhomogeneous traveling wave, if excited, can have a value of \( u_{\text{eq}} \) which can belong only to the interval where this fall occurs, i.e., \( u_{\text{eq}} \in W \) always.

The proof of this theorem is based on an investigation of the behavior of the integral on the right-hand side of Eq. (11).

One of the main implications of this theorem is that longitudinally inhomogeneous traveling waves can be excited only in a nonlinear medium whose nonlinearity \( (\alpha u)(u) \) has at least one interval of fall. One the other hand, longitudinally inhomogeneous traveling waves are prohibited in a medium with an increasing function \( \alpha u(u) \). This is the reason why excitation of longitudinally inhomogeneous traveling waves is prohibited for "positive" nonlinearities.

The "continuum problem" arises for all kinds of longitudinally inhomogeneous traveling waves. Therefore, the special issue in the theory of two-dimensional nonlinear waves is related to the "survival" of nonlinear transmitted waves in the case of arbitrary nonlinearity. Several theorems related to this problem were formulated in Ref. 4. In particular, a complete class of functions \( F(u) \) were found which allow for the "surviving" longitudinally inhomogeneous traveling wave, and it was proved that the principle of minimization of wave energy remains valid throughout this class.

5. EXPERIMENTS

For the purpose of describing the experiments, it is convenient to use a refractive index \( n \), rather than the susceptibility \( \kappa \). We write the index of the nonlinear medium as

\[
n(t) = n_0 + \Delta + n_2 I
\]

where \( n_0 \) is the index of the linear medium, \( \Delta \) is the mismatch between the refractive indices of the two media, which does not depend on the light intensity \( I \), and \( n_2 \) is the nonlinear index. In terms of the parameters used earlier in this chapter, we can write (for \( \Delta \ll 1 \), \( n_2 I \ll 1 \))

\[
\Delta = \frac{\Delta n_0}{n_0} \quad n_2 I = \frac{\epsilon_n E^2}{2\Delta n_0}
\]

The first experiment to observe the effects suggested by theory\(^3\) was reported in Refs. 7 and 9. In this experiment, the liquid \( C_2 H_5 \) was utilized as a nonlinear medium with positive nonlinearity \( (\epsilon_n > 0) \), and glass was used as the linear medium. The temperature regulation of
the cell served as a fine control of the difference $\Delta \omega$ between both susceptibilities required by the relationships in Eq. (3). The experiment is briefly described below following Refs. 7-9.

Experiments were performed with the apparatus shown in Fig. 6. The input pulse was generated by a mode-locked traveling-wave ruby laser. Internal Fabry-Perot mode selection was employed so that the laser output consisted of a train of $n = 1$ m pulses. The absence of internal structure in these pulses was verified with a streak camera. A single pulse was selected from this train and directed onto the setup shown in Fig. 6. A portion of the beam was sampled and directed with an optical delay of $\tau = 6$ ms onto a fast photodiode. The remainder of the beam was focused into the glass cell containing CS$_2$. The glass was chosen to have an index of refraction close to that of CS$_2$ at room temperature in order to satisfy the conditions in Eq. (3). The entire cell was placed in a temperature controlled holder and the temperature was adjusted so that for the 694.3 nm wavelength of the ruby laser, the index difference $\Delta n = 10^{-4}$. This corresponded to $\delta n = 2.6^\circ$ in the glass cell.

The reflected beam was monitored by the same photodiode used to monitor the incident pulse, and the diode output was displayed on a fast oscilloscope. The detector-oscilloscope combination had a measured response time of 320 ps. Fig. 7 shows the measured incident and reflected pulse shapes for an incident intensity slightly above the threshold for hysteresis. The dashed curve gives the results of calculations based on the plane-wave theory$^7$ (see Eq. (14)) and was fitted to the data by adjusting the ratio of the peak intensity to the threshold intensity $I_0 = |I_{th}|^2$, which corresponds to switching from a total internal reflection to partial

![Diagram of experimental setup](image)

**Fig. 6** Experimental setup. After Ref. 7.

![Graph of experimental measurements](image)

**Fig. 7** Experimental measurements of incident and reflected pulse shapes for $\omega_0/\omega_p = 0.707$. (a) The solid curve is the incident pulse, and the dots are the experimental measurements of the reflected pulse. The dashed line is the reflected pulse calculated from an incoherent plane-wave analysis for $\omega_0 = 1.125$ (see text). (b) The experimental data in (a) plotted in a way that demonstrates the optical hysteresis observed. After Ref. 7.
transmission state. The fit becomes less good as the peak input intensity is increased (see Fig. 8), however, the plane-wave theory still accurately predicts the threshold intensity \( I_T = \frac{I_0}{2} \) (Eq. (24) above) for the onset of switching from total internal reflection to transmission. Moreover, one can see from Figs. 7 and 8, that always\(^8\)

\[
I_T = \frac{I_0}{2}
\]

which is also consistent with the plane-wave theory prediction [see Eq. (25) as compared with Eq. (24)].

From the published value\(^9\) \( n_0 = 3 \times 10^4 \text{ (MW/cm}^2\text{)}^{-1} \) for Cs\( _2 \), the theoretical value of \( I_T = 8.1 \times 10^8 \text{ W/cm}^2 \) was computed based on Eq. (24) above. The experimentally measured value was \( I_T = 7.5 \times 10^8 \text{ W/cm}^2 \), in very good agreement with the theoretical value. In Fig. 7b, the same experimental data as in Fig. 7a are shown but replotted in the form of reflectivity versus input intensity. This form emphasizes the optical hysteresis observed. However, because of the limited time resolution of the detection system, and the fact that the experiments were only done for a single input pulse width, the observed hysteresis does not prove that the reflectivity is bistable.

Finally, the measured values of the threshold intensity \( I_T \) as a function of angle of incidence \( \phi \) are shown in Fig. 9. We see that the results are again in good agreement with the predictions of the plane-wave theory [Eq. (24) above].

Some of the experimental results show substantial disagreement with plane-wave calculations. For instance, for sufficiently large peak input intensities, the measured reflected pulse shapes deviate significantly from those obtained from calculations. It was believed and seemingly backed by computer simulations\(^10-13\) that at least some of the disagreements could be caused by the excitation of a nonlinear surface wave propagating along the interface with no decay, beginning from the point of reflection of the Gaussian beam. Indeed, it was shown\(^16\) that a nonlinear surface wave in the form of a stationary channel can propagate along the interface, traveling from \( s = -\infty \) to \( +\infty \), in the absence of any wave incident from either linear or nonlinear media. (Similar waves were discussed previously\(^14,15\) in application to plasmas.) However, using a general theory of the stability of solitons of a cubic-nonlinear Schrödinger equation,\(^16\) it was pointed out\(^17\) that if the surface nonlinear wave (in fact, a soliton solution)
propagates in the direction $x \rightarrow -\infty$ beyond the point where the incident Gaussian beam hits the interface, then the same soliton-like surface wave (albeit with a spatially shifted maximum) must exist even before this point, i.e. this wave can be observed even when $x \rightarrow -\infty$. This would be inconsistent with the formulation of the problem (indeed, it is presumed that at $x \rightarrow -\infty$, the entire energy of the field is concentrated in the linear semi-space); therefore the "semi-infinite" nonlinear surface wave is prohibited.

A computer simulation confirmed this result. Furthermore, it showed that when the intensity of a Gaussian laser beam exceeds a critical level, the total internal reflection state is drastically disturbed. The laser beam, instead of being totally reflected, breaks through the interface and propagates in the nonlinear material away from the interface. The intensity of the beam at this moment is sufficiently high for the beam to form a self-trapped channel, as shown in Fig. 10. If the incident intensity increases further, a second channel is formed.

It is natural to expect the formation of more and more channels as the intensity increases, which would correspond to a multichannel solution. The formation of each self-trapped channel is accompanied by a downward jump in the reflection coefficient. Since the self-trapped channels form right at the interface, resulting in a strong spatial modulation of the intensity at the interface, it is clear that this phenomenon cannot be described by the plane-wave theory, in which it is assumed that the transmitted field is a homogeneous plane wave. The simulations used a steady-state model, which is not capable of proving, or disproving, the existence of bistability, but it was postulated that a nonlinear interface cannot exhibit bistability, because of the lack of any memory mechanism to store the information on the previous field configuration. (This argument does not exclude bistable behavior for the case of other nonlinearities, such as either thermal or diffusion nonlinearities, that are either non-instantaneous and/or nonlocal).

In later experimental research, an "artificial" nonlinear medium (dielectric spheres suspended in a liquid) was used in order to observe switching at a nonlinear interface in the cw regime. The results of these observations were very close to the computer results. Fig. 11 shows the results of the computer calculations for a one-dimensional Gaussian beam. The experimental reflectivity data shown in Fig. 12 are in good qualitative agreement with this model. Perhaps more striking is the excellent agreement between the experimental data for the critical intensity for reflectivity "jumps" and the predictions of the Gaussian-beam model (Fig. 13).

For short observation times, hysteresis was observed. Only the upper branch of the reflectivity hysteresis characteristic was stable, however. The lower branch would decay to the upper branch with a surprisingly long time constant of $\sim 10^6$ times the response time of the nonlinear medium. The nature of this effect remains unclear, although it suggests, at least, that for any application with a pulsed mode of operation, the nonlinear interface will demonstrate...
Fig. 10 Perspective plot showing the results of numerical simulations of the behavior of a nonlinear interface for an incident one-dimensional Gaussian beam. This plot is for an input intensity such that a self-focused channel forms in the nonlinear medium. For this figure, from Ref. 13, the definitions of the $x$ and $z$ coordinates are interchanged from those used in the rest of this chapter.

Fig. 11 Reflectivity as a function of intensity for a nonlinear interface; one-dimensional Gaussian beam theory. After Ref. 20.
Fig. 12 Experimental plot of reflectivity versus incident intensity for $\theta = 0^\circ$. After Ref. 20.

Fig. 13 Critical intensity for reflectivity "jumps" as a function of incidence angle. After Ref. 20.
hysteresis. Indeed, the most recent experimental work\(^{18}\) due to essentially the same kind of media as in Ref. 20 and using a long pulse mode of operation, revealed very distinct hysteretic jumps. The authors of Ref. 56 have also experimentally observed the large nonlinear Geer-Hitchen effect suggested in Ref. 2 and obtained in a computer simulation.\(^{14}\) The Geer-Hitchen effect consists of the displacement of a reflected beam along the interface with regard to the point of incidence (see e.g. Ref. 57). It was shown in Ref. 13 that for intensities of the incident beam slightly lower than the critical intensity of formation of the first self-trapped channel, the point of reflection at the nonlinear interface moves very far away from the point of incidence as the incident intensity increases.

6. CONCLUSIONS

In this chapter, we have attempted to review the present status of the theoretical analysis of nonlinear interface phenomena, and to present some of the available experimental data. In general, the experimental results are in good agreement with theoretical predictions. The major area where uncertainty remains is the question of optical bistability. No clear evidence from either theory or experiment exists for the presence of two completely stable states when finite sized optical beams are involved.

There are many phenomena predicted by the theoretical analyses that have not yet been observed experimentally. This is primarily due to the difficulty of finding suitable nonlinear media. Recent studies of nonlinear coefficients in highly transparent optical glasses\(^{16}\) may lead to all-glass interface structures that will allow detailed experimental verification of many of the novel phenomena predicted by the analysis presented in this chapter.

7. ACKNOWLEDGMENTS

The work by A. E. Kaplan is supported by AFOSR.

REFERENCES

1. A E Kaplan, JETT Lett. 24, 114 (1976)
22. N Bloembergen and D S Peripha, Phys. Rev. 128, 606 (1962); N Bloembergen and J Ducuing, Phys. Lett. 6, 5 (1963);
44. V A Pomyakov and O V Bagdatsaryan, Radiophys. Quantum Electron. 21, 92 (1978)
50. A E Kaplan, JETP Lett. 9, 33 (1969)
52. M J Morais, C Y She and R L Curren, IEEE J. Quantum Electron. QE-11, 259 (1975)
57. Similar radiative emission from a nonlinear waveguide has recently been demonstrated in a computer simulation by E N Wright, G I Stegeman, C T Sutin, J V Moloney and A D Boardman, to be published
60. S Friberg and P W Smith, IEEE J. Quantum Electron. (to be published)