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Large Enhancement of the Sagnac Effect in a Nonlinear Ring Resonator and Related Effects

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Abstract

A fundamentally novel way is discussed to enhance the Sagnac effect and to obtain directionally asymmetric bistability by using a nonlinear ring resonator. These effects are based on nonlinear nonreciprocity induced by counterpropagating waves due to formation of nonlinear index grating. The enhancement of the Sagnac effect could be as large as $10^4 - 10^5$ which allows one to accordingly increase the sensibility of the passive resonant rotational sensors.

1. The Main Principles

It is well known that the Sagnac [1] effect provides a powerful way of measuring rotation rates with extreme accuracy – better than 10^{-4} x the earth's rotation rate [2]. It can be used for geophysical applications (e.g., for measurement of the earth's rotation, its precession, etc. [3]), and in general relativity (e.g., for measurement of the curvature of space [4]). Most important is its application as a rotation sensor, i.e., "optical gyroscopes" in intertial navigation systems [1,5] (e.g., in aircrafts and missiles).

We discuss here a fundamentally novel way to dramatically enhance the Sagnac effect in the passive ring resonator, which is based on nonreciprocity, induced in a nonlinear sample set in the resonator. The basic idea

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for this effect was first proposed by Kaplan and Meystre [6,7]. The proposed idea takes advantage of the different speed of propagation of counterpropagating waves of different intensities $|E_1|^2$ and $|E_2|^2$ in a nonlinear medium, which could be considered as nonlinearly-induced nonreciprocity. It is well known [6-10] that due to formation of a nonlinear index grating, the effective nonlinear susceptibilities ($\Delta\varepsilon_1^{NL}$ and $\Delta\varepsilon_2^{NL}$) of each of these waves are different provided that intensities of these waves are different. For instance, if a nonlinear material has a Kerr-like component of susceptibility,

$$\Delta\varepsilon^{NL} = \varepsilon_2 |E(x)|^2 \tag{1}$$

where E(x) is the total field at location x at the axis of propagation, then, in the presence of two counterpropagating waves, the effective nonlinear components $\Delta\varepsilon_j^{NL}$ (j = 1,2) for both of these waves are different:

$$\Delta\varepsilon_1^{NL} = \varepsilon_2\left(|E_1|^2 + 2|E_2|^2\right) \; ; \quad \Delta\varepsilon_2^{NL} = \varepsilon_2\left(|E_2|^2 + 2|E_1|^2\right) \tag{2}$$

provided that $|E_1|^2 \neq |E_2|^2$. Here, E_1 and E_2 label the right and left (or clockwise and counterclockwise) propagating field, such that total field E is

$$E(x) = E_1(x)e^{ikx} + E_2(x)e^{-ikx} \quad . \tag{3}$$

The nonreciprocal factors in Eq. (2) result from the formation of a nonlinear index grating which leads to the cross-interaction of E_1 and E_2, as can be shown from the following argument. The nonlinear part of the polarization is given by $P^{NL} \propto E\Delta\varepsilon^{NL}$. Using Eq. (3), in the case of Kerr-nonlinearity (1) this gives [6]

$$P^{NL} \propto E|E|^2 = \left(E_1e^{ikx} + E_2e^{-ikx}\right)\left[|E_1|^2 + |E_2|^2 + \left(E_1E_2^{*}e^{2ikx} + c.c.\right)\right] =$$
$$\propto e^{ikx} E_1\left(|E_1|^2+2|E_2|^2\right) + E_2\left(E_2^{\,2} + 2\,E_1^{\,2}\right)e^{-ikx} + \text{(rapidly varying terms)} \;, \tag{4}$$

so that the relevant polarizations for the clockwise and counterclockwise fields are as given in Eq. (2). We would like to stress two important facts with regard to this result. First, nonlinearly-induced nonreciprocity (2) is solely due to formation of a nonlinear index grating which is caused by the periodic term $[E_1E_2^{*}\exp(2ikx) + c.c.] \propto \cos 2kx$ in $|E|^2$. If this periodic grating is washed out by some process (e.g., diffusion), then nonreciprocity in Eq. (2) should decrease. Second, cross interaction (2) caused by the Kerr-nonlinear index grating, does not lead to the mutual back reflection of both of the counterpropagating waves at the grating, as has been explicitly shown in Ref. [9]. Hence, the effect discussed here is caused solely by the nonlinear change of optical paths, not by the induced reflection. The linear Sagnac effect is caused by the linear nonreciprocity (i.e., different optical paths of two counterpropagating waves), which is due to rotation of the waveguide system, e.g., a ring interferometer. In a resonator filled with a nonlinear medium, the optical paths are further varied as functions of light intensity. If this nonlinear change is the same for both waves, it has no influence on the Sagnac effect. However, if the nonlinear changes of optical paths are different for the left and right propagating waves, Eq. (2), it can lead to considerable increase in the total nonreciprocal effect, provided that intensities of these waves are different.

Suppose now that the ring resonator is pumped in both directions by two incident beams of the same intensity $|E_{in}|^2$ (and frequency ν) (see Fig. 1).

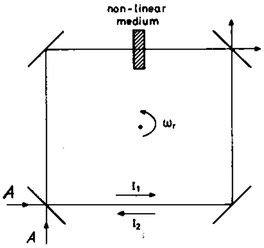

Fig.1 Rotating ring resonator with nonlinear medium and two pumping beams of equal intensity.

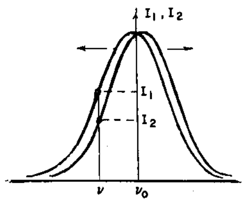

Fig.2 The Sagnac splitting of resonance for clock- and counterclockwise propagating waves.

In the absence of rotation, the intensities of the waves excited inside the resonator should be the same, $|E_1|^2 = |E_2|^2 \equiv |E_0|^2$. Suppose also that the laser frequency is set on the side of the resonant curve (Fig.2) in the vicinity of one of the eigenfrequencies of the resonator ν_0. Then, in the presence of rotation, the Sagnac effect causes a different change of the intensities of the waves. This is because of the sharp dependence of each of the intensities on Sagnac detuning at the slope of the resonant curve: one of the intensities decreases while the other increases. Therefore, the initial small difference of intensities provided by the usual Sagnac effect can be used to generate a large additional difference via the nonlinearly-induced nonreciprocity of the optical paths of the counterpropagating waves.

The initial phase shift between both of these waves $\Delta\phi$ due to the linear Sagnac effect is $\Delta\phi_0 = 4S\omega_r Q/cL$, where S is the area of the ring resonator, Q and L are its finesse and total optical path, ω_r is the rotation rate. By using a resonator filled by a nonlinear material and by tuning the frequency of the light to the proper slope of the resonant curve, one can obtain a large increase of the measured Sagnac phase shift, $\Delta\phi = \eta\Delta\phi_0$, where η is the factor η of nonlinear enhancement (see Section 2 below). The factor of enhancement η could be as large as $10^4 - 10^5$, which allows one to accordingly increase the sensitivity of the passive resonant rotational sensors based on the Sagnac effect.

Under some conditions, the factor of enhancement becomes infinite which results in some other new effects due to nonlinear nonreciprocity in the ring resonators (besides enhancement of the Sagnac effect), in particular, directionally asymmetrical optical bistability [7]. Inside a domain whose boundary corresponds to $\eta = \infty$, the symmetrical waves generated in the ring resonator become unstable even without any rotation. This instability leads to damping of one of the propagating waves and to an increase of intensity of another counterpropagating wave. This asymmetrical excitation (under symmetrical pumping) results in a novel kind of optical bistability which provides another opportunity to obtain spatial (directional) switching of radiation, in addition to some previously proposed and experimentally observed spatial nonlinear switching and bistability (at nonlinear interfaces [11,12], and in mutually self-acting beams [13]).

2. Enhancement of the Sagnac Effect

We consider a rotating passive ring resonator filled with a nonlinear medium. The system is pumped in both directions by two incident beams of the same frequency ν and amplitude E_{in}. Taking the frequency ν of the driving field to be close to one of the eigenfrequencies ν_0 of the resonator, the steady-state amplitude E_j of the field j (j = 1,2) is

$$E_j = \frac{(\gamma_c/\gamma)E_{in}/\sqrt{T}}{1 + i\gamma^{-1}\left[\nu-\nu_0 + (-1)^j\omega_s + \nu_0\Delta\varepsilon_j^{NL}L_N/L\right]} \quad . \tag{5}$$

Here, T is the mirror transmittivity, L the total optical length of the resonator and L_N the optical length of the nonlinear sample. $\gamma_c = cT/L$ is the empty cavity bandwidth, and γ_s gives the linear losses of the medium inside the cavity, so that $\gamma = \gamma_c + \gamma_s$ gives the total bandwidth of the system, $k_o = \omega_o/c$, and

$$\omega_s = 4S\omega_r k_o/L \tag{6}$$

is the Sagnac frequency shift, where S is the area of the ring resonator and ω_r its rotation rate. Note that Eq. (5) is an approximate form valid for the good finesse case, that is, $\gamma \ll c/L$. We introduce the dimensionless frequencies $\Delta = (\nu - \nu_0)/\gamma$ and $\Omega = \omega_s/\gamma$, and the dimensionless intensities

$$I_j = \varepsilon_2|E_j|^2 L_N\nu_0/L\gamma \qquad (j = 1,2) \quad ,$$

$$A = \varepsilon_2|E_{in}|^2 L_N\nu_0\gamma_c^2/L\gamma^3 T \quad , \tag{7}$$

so that A corresponds to the driving field, and I_j to the generated waves. (Note that I_j, A, and Δ change sign for $\varepsilon_2<0$.) The equation for the intensity I_j is readily obtained by multiplying Eq. (5) by its complex conjugate:

$$A = I_j\left\{1 + \left[\Delta + (-1)^j\Omega + I_j + 2I_{3-j}\right]^2\right\} \quad . \tag{8}$$

We assume that in the absence of rotation the intensities I_1 and I_2 are equal (symmetrical regime). That is $I_1 = I_2 = I_o$, where I_o satisfies the equation

$$A = I_o\left[1 + (\Delta + 3I_o)^2\right] . \qquad (9)$$

We also restrict ourselves to the most important case of small rotations ($\Omega << 1$), so that we can assume that the perturbations $\xi_j = I_j - I_o$ of both waves are small ($\xi_j << I_o$). Under these conditions, we immediately find the solution for ξ_j:

$$\xi_1 = -\xi_2 = (\eta - 1)\Omega/2 \quad ,$$

where

$$\eta = \frac{1 + (\Delta + 3I_o)^2}{1 + (\Delta + 3I_o)(\Delta + I_o)} \quad . \qquad (10)$$

Therefore, the total phase difference between the two counterpropagating waves is given by $\eta\Omega$ instead of Ω. Hence, the result of the nonlinear medium is to scale the Sagnac effect by a factor of η. Eq. (10) is an implicit form for η, and I_o must still be determined from Eq. (9). One expects η to be much larger than unity for values of the driving field and detuning such that the system is near the onset of an instability.

Calculations based on Eqs. (9) and (10) show that the boundary of the unstable regime ($\eta \to \infty$) is given by

$$A = (2/3)\left[-\Delta(3\Delta^2 - 5) \pm (3\Delta^2 - 1)\sqrt{\Delta^2 - 3}\,\right] . \qquad (11)$$

This domain of instability in the plane of parameters A and Δ is shown in Fig. 3 as curve A_1. As shown in Section 3 below, this instability leads to a new steady-state regime such that $I_1 \neq I_2$ (non-symmetrical regime) in the absence of rotation of the ring interferometer. The detuning threshold of this instability occurs for $\Delta_{th} = -\sqrt{3}$, which corresponds to $A_{th} = 8/\sqrt{3}$.

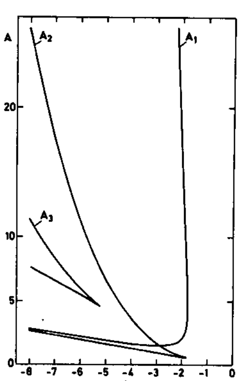

Fig.3 Domains of operation in the plane of parameters A and Δ. Curve A_1 defines the domain of instability of the symmetrical solution; A_2 the domain of bistability of the symmetrical solution; and A_3 the domain of multistability of the asymmetrical solution

We find that the enhancement factor η increases dramatically as one approaches the instability boundary from the symmetrical regime of operation, i.e., as one decreases Δ. Note that for $\Delta > \Delta_{th}$, the regime of operation for $\Omega = 0$ is symmetrical and stable for any value of the driving field.

In Fig. 4 we show the enhancement factor η as a function of the driving field A and for various detunings $\Delta \geq \Delta_{th}$. These curves were obtained by solving Eqs. (9) and (10) consistently. As expected, η becomes infinite for $\Delta = \Delta_{th}$ and $A = A_{th}$. It follows from Eq. (10) that for a fixed detuning, the highest enhancement is given by

$$\eta_{max} = 3/\left(2 - \sqrt{\Delta^2 + 1}\right) \quad . \tag{12}$$

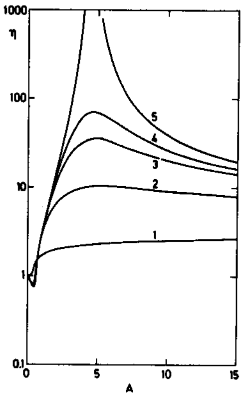

Fig.4 Enhancement factor η as a function of the dimensionless pump intensity A for various values of the dimensionless frequeny detuning Δ: 1) $\Delta=0$, 2) $\Delta= -\sqrt{3}(1-0.2)$, 3) $\Delta= -\sqrt{3}(1-2/35)$, 4) $\Delta= -\sqrt{3}(1-1/35)$, 5) $\Delta= -\sqrt{3}$

In the vicinity of the threshold ($\Delta - \Delta_{th} << |\Delta_{th}|$), Eq. (12) reduces to $\eta_{max} \cong 2\sqrt{3}/(\Delta - \Delta_{th})$. The corresponding amplitude of the driving field is

$$A_{max} = -2(\Delta^2 + 1)\left(\sqrt{\Delta^2 + 1} + 1\right)^2/3\Delta^3 \quad . \tag{13}$$

It is interesting to note that even far from the instability threshold, the enhancement remains very large. In the limit $A \to \infty, \eta \to 3$, independently of Δ. We also remark that the enhancement is only weakly sensitive to changes of the driving field, which indicates the possibility of using pump lasers with relatively high intensity fluctuations.

Under reasonable requirements for stability of parameters of the resonator and laser radiation, the proposed system has the potential to achieve enhancement as large as $10^3 - 10^4$, which allows one to accordingly increase the sensitivity of the passive rotation sensor. For example, provided that the frequency stability of the resonator and laser radiation is about 10^{-9}, $Q \sim 10^3$, and the amplitude stability of the laser [14] is about 10^{-3}, Eqs. (10,12) show th feasibility of obtaining $\eta \sim 10^4$ with stability $\sim 10^{-2}$.

One of the most important facts about nonlinear enhancement of the Sagnac effect is that due to the relatively slow time scale of its conventional applications (usually, it is more than a few msec), one can use huge but slow nonlinearities, recently discovered and studied in the optical and IR ranges, for example, in InSb.[15] We illustrate this with the following example. We assume small mirror transmittivity ($T \cong 0.1$), neglect the internal losses ($\gamma_S \ll \gamma_C$), and take advantage [15] of the large $\chi^{(3)}$ in InSb ($\chi^{(3)} = 10^{-2}$esu). Considering $L_S \cong 100$ µm and $k_0 \cong 1.2 \times 10^4$ cm^{-1}, we obtain, with the threshold condition $A_{th} = 8/\sqrt{3}$, the required incident field, $|E_{in}|_{th} \cong 45$ V/cm. This corresponds to an incident power of about 50 mW/mm^2, which can readily be achieved in the cw regime.

3. Directionally Asymmetrical Bistability

As was already mentioned above, the cross-interaction of two counterpropagating waves via the nonlinear index grating that they generate could result in instability of the symmetric regime of propagation, even without any rotation of the ring resonator. This instability leads to the non-symmetric regime, in which the amplitudes of the two counterpropagating waves become different, while the ring resonator is symmetrically pumped by two beams of the same intensity and opposite directions. We immediately note that this effect could not occur in the case of single beam pumping (which is characterized by symmetrically optical bistability [16]). Direction of the dominantly generated wave could be clockwise as well as counterclockwise under the same conditions. This corresponds to the situation which could be described as directional optical bistability. Optical bistability has been the object of intense activity over the last few years [17]. Up to now, the bulk of the research has been concerned with intensity switches in a nonlinear Fabry-Perot resonator (except some nonresonator devices [11-13]), or in ring cavities pumped by a single beam. The existence of the directionally asymmetric regime is of great interest for applications where directional switching of light is required.

We consider again the same nonlinear ring resonator (Fig. 1), now without any rotation ($\omega_r = 0$; $\Omega = 0$). In such a case, the equation (8) for the waves, intensities I_j ($j = 1,2$) can be written in the form

$$A = I_j \left\{ 1 + [\Delta + I_j + 2I_{3-j}]^2 \right\} \quad . \tag{14}$$

We consider two different solutions, the symmetrical and asymmetrical ones. In the symmetrical case, ($I_1 = I_2 = I_0$), the intensities satisfy Eq. (9). The intensity I_0 as a function of the incident intensity A is shown in Fig.5 (symmetrical branch) for various values of the detuning Δ. As can be seen from Figs.5b, c, d, the symmetrical regime exhibits hysteresis for sufficiently large values of Δ and A. By requiring that $dI_0/dA = \infty$ at the points of hysteretic jump, we find readily from Eq. (9) that the domain of usual symmetrical bistability is given in the plane of the parameters A and Δ by

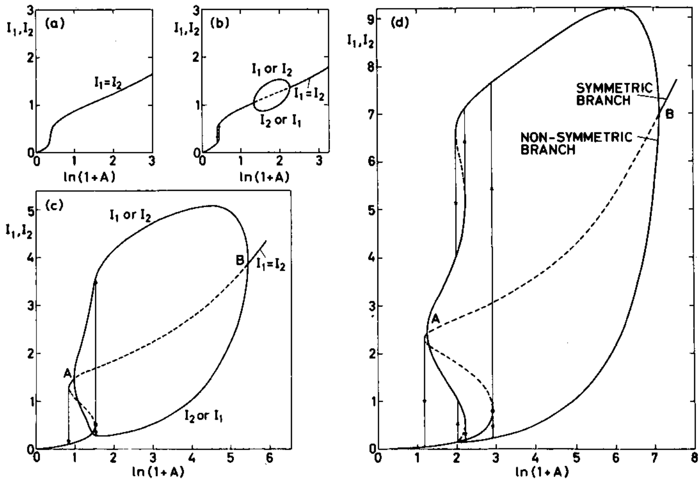

Fig.5 Normalized intensities I_1 and I_2 as a function of the dimensionless pump intensity A. The dashed parts of the curves indicate the regions of instability. The curves labeled by $I_1 = I_2$ give the symmetrical regime, and the points A' and B' the crossings between symmetrical and asymmetrical regimes. The four figures are for various values of the dimensionless detuning Δ. (a) $\Delta = -1.65$ (b) $\Delta = -1.8$ (c) $\Delta = -4$ (d) $\Delta = -7$

the relation

$$A = (2/81)\left[-\Delta(\Delta^2+9) \pm (\Delta^2-3)^{3/2}\right] . \tag{15}$$

This domain in shown in Fig.3 as curve A_2. The detuning threshold for the onset of symmetrical bistability is given by $\Delta_2 = -\sqrt{3}$, which corresponds to a driving intensity $A = 8/9\sqrt{3}$. However, it can easily be shown by a linear stability analysis of Eq. (14) that under appropriate conditions, the symmetrical regime determined by Eq. (9) becomes unstable. The domain of instability of the symmetrical solution is given by Eq. (11), which is shown as curve A_1 in Fig.3. The detuning threshold for this regime is $\Delta_1 = -\sqrt{3}$ which is equal to Δ_2, but corresponds to the larger pump intensity $A = 8/\sqrt{3}$. Inside this domain, the stable behavior of the system is characterized by the fact that I_1 and I_2 are unequal. For $I_1 \neq I_2$, the two coupled third-order Eq. (14) can be reduced to a single third-order equation for $S = I_1 + I_2$,

$$A = \left[1 + (S+\Delta)^2\right]\left[2\Delta + 3S\right] , \tag{16}$$

which can be solved readily for any values of A and Δ. The separate values of I_1 and I_2 can then be obtained via the equation

$$A = P(2\Delta + 3S), \qquad (17)$$

where $P = I_1 I_2$.

The asymmetric solution is shown in Figs.5b,c,d. Note that I_1 and I_2 play completely symmetrical roles in Eqs. (16,17), so that which of them will become larger is determined by noise or slight initial asymmetries in pumping.

It is interesting to point out that the asymmetrical regime can itself become multivalued for large enough Δ and A, as seen in Fig.5d. Applying the same criteria as in the symmetrical case ($dI_j/dA = \infty$, $j = 1,2$) we find that the domain of multistability of the asymmetrical branch is given by

$$A = (2/3^5)[-\Delta(\Delta^2 + 81) \pm (\Delta^2 - 27)^{3/2}] \ . \qquad (18)$$

which is shown in Fig.3 as curve A_3.

4. Conclusion

The nonlinearly induced nonreciprocity of counterpropagating beams of light in the nonlinear ring resonator can be used to dramatically enhance the Sagnac effect and therefore to increase sensitivity of the passive optical rotational sensors. This enhancement of the Sagnac effect could obviously find applications in cases where high sensitivities are required (e.g., optical tests of general relativity). On the other hand, this effect provides a direct way of studying experimentally non-local interactions of light with matter, since it is due solely to the formation of a nonlinear index grating, which, in turn, can be formed if the nonlinearity depends only on the light intenstiy at a given point (local interaction). If the index grating is washed out by some fast process (for instance, diffusion in gases), the non-reciprocal factor 2 in Eq. (2) is replaced by $1 + \alpha$, where in the general case $0 \leq \alpha \leq 1$. This leads to a reduced enhancement factor $\eta_\alpha = 1 + \alpha(\eta - 1)$. Thus, the measurement of η_α provides a novel spectroscopic method of analyzing non-local interactions (non-reciprocal Sagnac spectroscopy).

The effect of directionally asymmetrical bistability is based as well on nonlinear nonreciprocity, and could be applied for directional switching, e.g., in optical signal processing. In addition, this effect should be very useful in connnection with enhancement of the Sagnac effect. Observation of asymetrical bistability will immediately help to locate the zone of the large enhancement which should occur in the vicinity of the onset of this bistability.

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